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THE

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VISUAL OBSERVATIONS OF HALLEY'S COMET IN 1910

By E. E. BARNARD

Considering its great brightness, and the extraordinary phenomena presented by other comets of recent years which at most only attained the faintest naked-eye visibility, Halley's comet at its return in 1910, though a brilliant and interesting object to the naked eye—especially in the month of May—was, nevertheless, a disappointment when considered from a photographic standpoint. It is safe to say that it did not give us any new information concerning these strange bodies. Photographically, its light was relatively slow in its action on the sensitive plate, and there were few or none of the remarkable phenomena shown by Brooks's comet of 1893, which was faintly visible to the naked eye for about one day, and by Morehouse's comet of 1908, which just attained naked-eye visibility for a couple of days. Had it not been for the previous comets, however, the numerous photographs obtained of it would have put Halley's comet in the first rank among the records of these bodies. While it lacked much in interest as seen with the eye of the sensitive plate, to the human eye it left a lasting impression which, added to its long life-history of more than two thousand years, made it, at its return of 1910, perhaps the most interesting comet of history. The apparent length of its tail when nearest the

earth (120° or more) was probably the greatest on record,[1] though the actual length was much exceeded by many previous comets. As seen from this observatory it was visible to the naked eye from April 29 to June 11, which was not an excessive duration of visibility. With the 40-inch telescope its visual appearance extended from September 15, 1909, to May 23, 1911, which, though a long period, has been exceeded by several comets that never attained naked-eye visibility.

HALLEY'S COMET FROM A POPULAR POINT OF VIEW

In this place it may be well to say a word or two on the popular side of this return of Halley's comet.

It is unfortunate that the newspapers and the general public were so greatly disappointed in the comet—unfortunate from the fact that the general impression left by such reports would be exceedingly misleading when comparing the present return with descriptions of its appearance in earlier times. It was unfortunate, also, from the further fact that even astronomers sometimes have a sentimental side. It would have been a gratification to know that everyone who saw this wonderful object saw it with the same spirit of elation and wonder—one would almost say veneration—with which the average astronomer regarded it. This was, at least, the feeling of the present writer when he looked at this beautiful and mysterious object stretching its wonderful stream of light across the sky.

The great cities that have grown up since 1835, and the smoke and electric lights of today completely robbed the comet of its glory when seen by dwellers in and near the centers of population. The newspapers had excited the public pulse to a high pitch by glowing and sensational accounts of what the comet would do and what it would look like, and had thus raised expectation beyond all reason. When these expectations failed, purely because of local

[1] According to Ellery, comet I 1865, for several days in January, had a tail 150 degrees long (*Monthly Notices*, **25**, 220). I find, however, that this great length is simply a printer's error of 150 degrees for 15 degrees. See a note by Ellery in *Astronomische Nachrichten*, **64**, 219 (same date as the one in *Monthly Notices*), where he gives the length as 15 or 16 degrees. This smaller length is verified by other southern observers.

conditions, it was not possible for them to pile enough contumely upon the comet and upon the heads of those who had made no prediction whatever as to what the comet might really look like. Were such records as these the only ones to depend upon for comparison at future returns, it would indeed be unfortunate. In reality, to those who under favorable conditions saw the comet at its best at the return of 1910, and who would have been justified in making any prediction, it far exceeded the most sanguine expectations in the remarkable display it presented to us.

There was one fact which was brought forth by the comet with startling vividness. It showed that the superstitious terror formerly attending the appearance of a great comet is by no means dead in the human breast. Cases of this kind developed all over this country and abroad—from the stopping-up of keyholes and cracks in doors and windows in Chicago (according to the daily papers) to keep out the deadly comet gases, to the manufacture and sale, among the negroes of the South, of "comet pills," which were supposed to ward off the evil effects of the comet.

The "comet gas" scare seemed to be directly due to the incautious and unwarranted statements of one or two men of science who had painted in rather vivid language the direful effects of breathing the deadly cyanogen gas, which had been shown to exist in the tails of some comets.

Such being the case in our present enlightened day, it is easy to understand how terrifying the comet must have been in former times, even if its display then was no more striking than in 1910. In the calm of the spring night, at a time when one is easily impressed with a mystery that is not present in the day, the comet, with its weird streamer of light reaching far into space, was well fitted either to impress or to terrify the observer, just as his mental temperament might suggest. In the Dark Ages, when the mission of these dread bodies was unknown, and when everything in nature seemed to possess a spirit for good or evil—and mostly for evil—there is little wonder that the unheralded advent of a great comet should inspire, at the least, an uneasiness in the minds of those who saw it. The writer was strongly imbued with this thought on several nights while observing Halley's comet when in its most

impressive stage, and it would not have taken much imagination to have endowed it with a guiding spirit. With the enlightenment of today, however, one could see nothing in it that would disquiet or terrify, but rather that which raised a sense of extreme pleasure and wonder at the magnificent mystery it presented.

POSSIBLE ENCOUNTER OF THE EARTH WITH THE TAIL OF THE COMET

In connection with this account of Halley's comet and its near approach to the earth, it may be appropriate to add some remarks on the probable encounter of the earth with a portion of the tail at, or closely following, the time the comet transited the sun. Indeed, it seems more than probable that the earth actually did encounter one of the branches of the tail—the southern branch—on, or about, May 18 or 19, and more probably on the later date. There is also a suspicion that the influence of this encounter (if such there was) on our atmosphere was apparent for months afterward.

The double tail seen here on the nights of May 17 and 18, the lower, and probably larger branch of which widened toward the southeast horizon, involved the ecliptic, as will be seen by the diagram on p. 389, and without doubt extended beyond the earth. There are strong chances that the earth passed through this part of the tail about May 19. That the tail was long enough to reach to the earth is shown by the fact that as late as May 25 its length (54°) was over 30 million miles, or twice the distance of the comet at its nearest approach to us on May 18.

With the exception of a sketch by Miss Mary Proctor in New York City and a newspaper account by Professor D. P. Todd of Amherst (whose observation seemed to refer to May 16), I have seen no reference from northern observers to the second, fainter and broader tail shown in my drawings of May 17 and 18 south of the bright beam and separated from it by a distinct dark space perhaps 10 degrees wide. In Plate X I have tried to show as accurately as possible the appearance of the tails and their exact location among the stars on these two dates. The head of the comet was, of course, invisible below the horizon. This feature (the broad, faint, southern tail) seems to have been generally overlooked by observers in the Northern Hemisphere. It is, how-

ever, well shown in drawings made in South Africa by Innes and others at the Transvaal Observatory (now the Union Observatory). See *Circulars 3* and *11* of that observatory. It is also shown in a drawing made by Dr. Frank C. Cook, United States Navy, at Bahia Blanca, Argentine Republic, on May 19, 1910, at 5 A.M. In the South African sketches the south tail is generally shown fainter and very much broader, which agrees with my drawings. In my drawings the north edge of the south branch is well determined, but the south edge of it is evidently lost in the zodiacal light, which fills out the space to the southeast horizon.

Professor C. D. Perrine, at Cordoba, Argentine Republic, calls attention to and describes this second and broader tail (*Astronomical Journal*, **26**, 145).

One would have expected considerable parallax in portions of the tail on May 17 and 18. A comparison of the South African drawings with mine, however, does not show any parallax, at least none greater than the uncertainty of the drawings themselves.

During the first part of the night of May 18, as will be seen by the notes, the sky was normal. It is probable that the slight mistiness mentioned on that date was in no way connected with the presence of the comet. The slight aurora, also, was nothing out of the ordinary, and certainly had nothing to do with the comet. In the latter part of the night, when the moon had set, the sky seemed to be free from any decided mistiness to the north of the comet's tail. At the same time the southern and fainter branch seemed to spread its effect over the southeast horizon, but there was nothing especially suggestive in its appearance.

The forenoon of May 19, however, developed peculiarities that were very suggestive (*Astronomische Nachrichten*, **185**, 229, 1910). Briefly, these consisted of a peculiar iridescence and unnatural appearance of the clouds near the sun and of a bar of prismatic light on the clouds in the south. This, combined with the general effect of the sky and clouds—for the entire sky had a most unnatural and wild look—would have attracted marked attention at any other time than when one was looking for something out of the ordinary. The sky had been watched carefully during the forenoon of this date but nothing unusual had appeared until close to noon, when the

conditions became abnormal, as stated above. Of course this unusual phenomenon, if seen only at one place, might be considered a coincidence, but something similar was reported on that date at other widely distant places. (See *Transvaal Observatory Circular*, No. 3, p. 19)

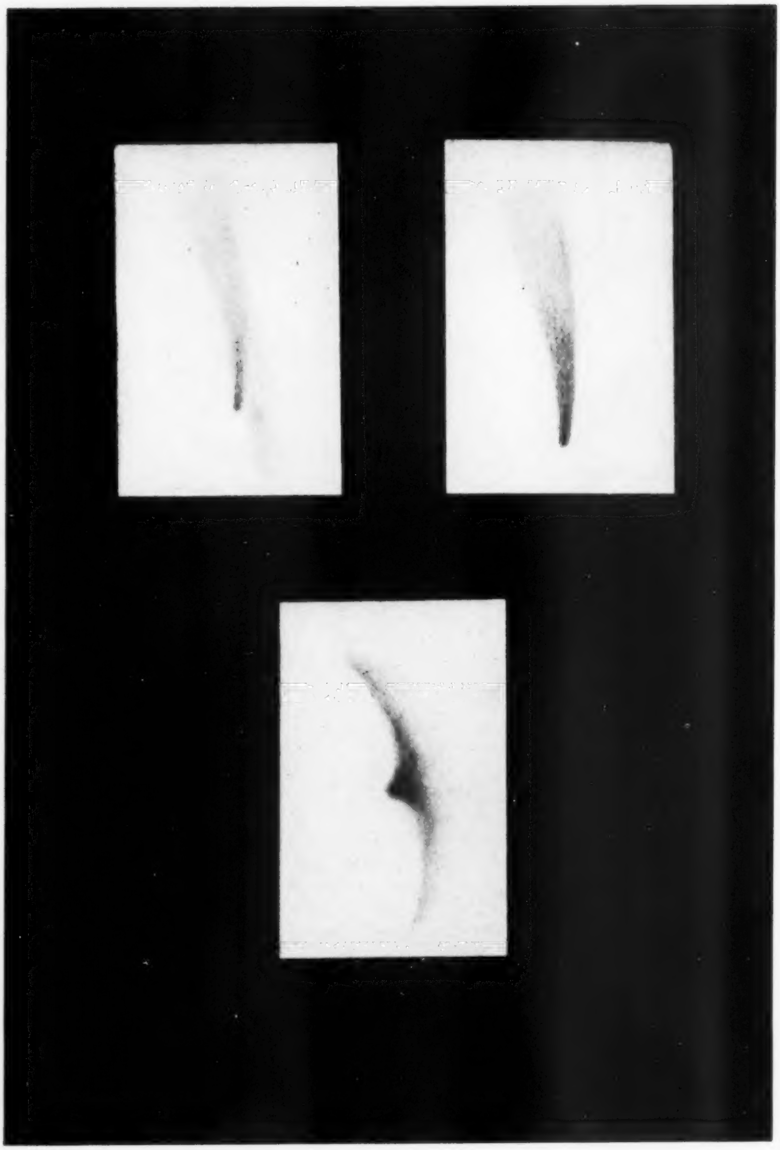
The most suggestive phenomenon, however, was apparent later on, in June and for at least a year afterward. It was first noticed here on the night of June 7, 1910, and consisted of slowly moving strips and masses of self-luminous haze which were not confined to any one part of the sky. I have given an account of these singular features in the *Proceedings of the American Philosophical Society* for May-June 1911. It is true that these peculiarities might in some way have been of auroral origin, but this I do not think is probable, for they do not seem to resemble in any way, either in position or in appearance, any auroral phenomena with which I am familiar. Apparently nothing of the kind has again been visible here since September of 1911. At the same time it is also true that a similar absence of essentially all auroral effects has been very marked here, during the same period. This luminous haze had not been noticed by me in past years previous to 1910—especially in those years in which I was almost constantly out at night comet-seeking.

I would be more disposed to believe that this phenomenon of luminous haze had some connection with the near approach of Halley's comet to the earth were it not for the fact that apparently a similar phenomenon was recorded by Mr. Backhouse at Sunderland, England, through many years (see *Publications of West Hendon House Observatory*, No. 2, p. 109, 1902). Mr. Backhouse's descriptions show that the phenomena seen by him were perhaps of a similar nature to those seen here in the fall of 1910. It is probable, therefore, that this luminous haze was in no way connected with the close approach to us of the tail of Halley's comet. Nevertheless, a record should be made here of the phenomenon for the benefit of posterity. These observations by Mr. Backhouse were not known to me when my paper was prepared for the Philosophical Society.

THE COMET WITH THE LARGE TELESCOPE

Observations of the physical appearance might have been very interesting if it had been possible to follow the comet closely with

PLATE VII



Drawings of Nucleus and Appendages
Upper figures, May 3; lower figure, May 4

the large telescope. It was necessary, however, for it to have attained a considerable altitude before it could be seen with that instrument. As it was, the comet could be observed with the large telescope only after dawn, when the nucleus and its brighter appendages alone could be seen. The smallness of the field ($5'.5$) and the great power of the telescope would have militated much against its successful use. From the clouded condition of the sky very few observations could be made during the morning visibility in April and May of 1910. A few rather unsatisfactory views were had, mainly when observing the comet for position later, in the last of May and in June. The most interesting observations, however, were obtained on the mornings of May 4 and 5, when the comet was watched in the coming daylight as it faded from view.

On May 3 (astronomical date), after the exposures with the Bruce telescope, the comet was observed with the 40-inch. Its aspect in the large instrument was rather singular. At first there were two wings to the nucleus, the southern of which was the brighter. The northern one, indeed, was so much fainter that it gave the nucleus and its appendages a very unsymmetrical appearance. Daylight soon blotted out the northern wing, leaving the nucleus with the southern one alone visible. It then very greatly resembled the naked-eye appearance of a great comet, with nucleus and tail. The accompanying sketches show the nucleus and wings as seen in the 40-inch telescope just before dawn obliterated the northern wing, and at 16^h20^m when only the southern wing and nucleus were visible (upper two sketches of Plate VII).

On May 4 at 16^h15^m, with the 40-inch telescope the nucleus and its appendages were more symmetrical. While on May 3 the matter was nearly all on the southern side of the nucleus, it was evenly distributed on May 4 (see lower sketch of Plate VII).

VELOCITY OF THE PARTICLES OF THE TAIL

Of the physical phenomena presented by the comet, the most interesting was shown on June 6, 1910. On that date a long receding mass appeared in the tail. This seemed to be a disconnected streamer. From photographs made here with the Bruce telescope, at Honolulu by Mr. Ellerman, and at Beirut, Syria, by Mr. Joy, the writer obtained the results shown in Table I for the

motion of the object and hence, also, for the motion of the particles forming the tail (*A.N.*, **186**, 11, 1910). At this time the recession of the comet's head from the sun was 16.6 miles (26.7 km) per second.

TABLE I

Station	Interval	Hourly Motion	Recession per Second			
			From Comet		From Sun	
			Miles	Km	Miles	Km
Y.O.-Honolulu	4^h25	3'.60	23.1	37.2	39.7	63.9
Y.O.-Beirut	15.15	5.17	33.1	53.3	49.7	80.0
Honolulu-Beirut ...	10.90	5.78	37.3	59.7	53.9	86.4

These results show a decided acceleration in the motion of the mass, which in the last two photographs amounted to an increase of 14 miles, or 22 km, per second. It should have been stated, however, that some uncertainty exists in the results owing to the possible change in the form of the mass. In all cases the end nearest the comet's head was measured, but this end itself may have shortened by dissipation of its material, and thus produced an apparent motion larger than the real one.

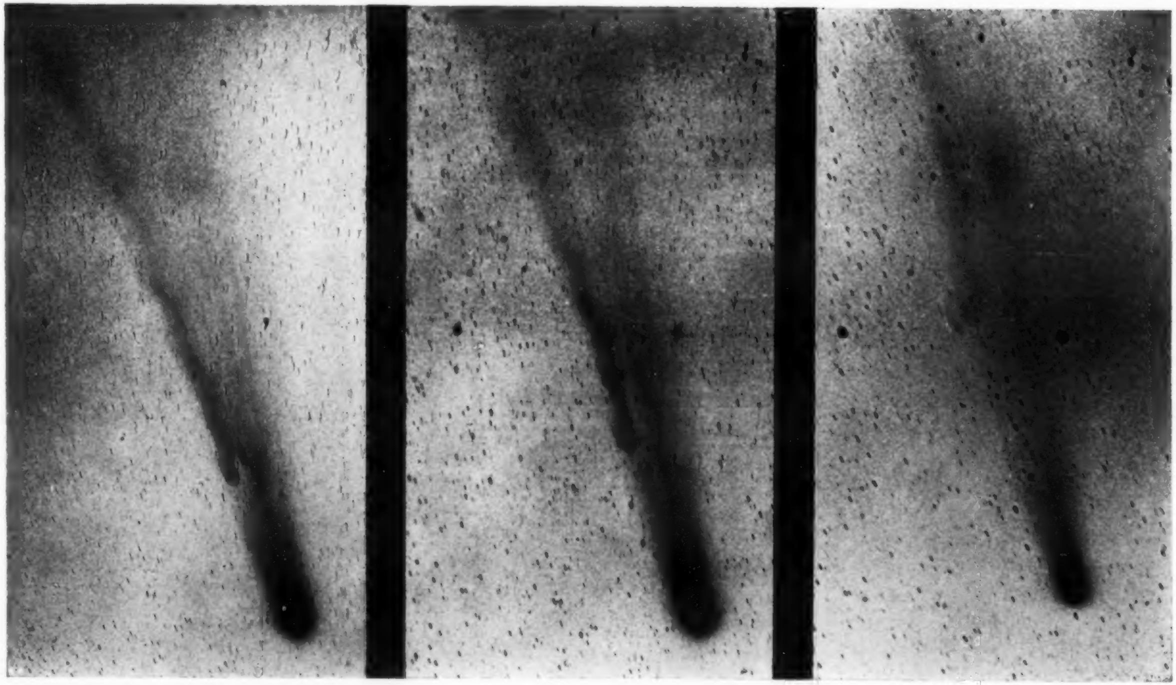
I have combined these three photographs (in the negative form) in Plate VIII. They are herewith presented so that the reader may judge for himself of the probability of any change in the actual form of the end of the mass.

Plate IX is a reproduction of the photograph of May 4. Attention has been called in the notes to the fact that heavy smoke from the power-house was drifting over the comet throughout the exposure of this plate, and that in effect it must have cut down the actual exposure-time by one-half. From this cause the full width of the tail is perhaps not shown.

It was unfortunate that May 4 and 5, the only two good mornings on which the whole comet was visible, were both spoiled by the smoke from the power-house that was driven south, directly over the comet, by a heavy north wind.

The tail of the comet on May 29 (Plate XI) shows considerable structure, which is more or less lost in the reproduction.

PLATE VIII



Yerkes: June 6, 15^h8 G.M.T. Honolulu: June 6, ~~18^h5~~ G.M.T. Beirut: June 7, 7^h0 G.M.T.

Discarded Tail Receding from Halley's Comet

$20^h.1$

PLATE IX

North

10-inch Lens. May 4, 21^{h} 2^{m} G.M.T. Exposure 40^{m}

Scale: 1 cm $= 0^{\circ}.41$

Thanks are due to Mr. Leon Barritt, editor of the *Evening Sky Map*, for the loan of the half-tone block made from my photograph of June 6 (Plate XII).

One rather striking feature of the tail during the last stages of its visibility was that the star 87 *Leonis* remained in it or close to it for a long time. The tail seemed to be slipping eastward by or over the star. This, of course, was due to the motion of the comet, and the changing position of the sun.

THE RETURN OF THE COMET AND ITS EARLY APPEARANCE IN THE LARGE TELESCOPE

During the fall and winter of 1908–1909, the writer made a careful search for Halley's comet, both photographically with the Bruce telescope, and visually with the 40-inch. At that time it was too faint for either instrument. As if in acknowledgment of the discovery of photography since its last return in 1835 and the wonderful progress made in the application of that science to astronomy, the comet was destined to be seen first by the sensitive photographic plate. It was actually discovered, photographically, by Dr. Max Wolf, with the 30-inch reflecting telescope at Heidelberg, Germany, on September 11, 1909. The first visual observations of the comet were made by Professor S. W. Burnham with the 40-inch telescope of the Yerkes Observatory, on September 15, 1909.

At my first observations with the large telescope, beginning September 17, 1909, the comet was a small and rather faint speck of light, very much like a faint stellar nebula, of which there are so many in the sky. It was by no means at the limit of the great telescope, and under favorable conditions could have been seen much earlier with that instrument. The increase in brightness was not very rapid and as late as the last observations in February 1910, before the comet passed behind the sun, it gave very little promise of the splendid display it was destined to make later, in the month of May. Its reappearance from behind the sun in the morning skies of April and May could not have been under more unfortunate circumstances for observation at the Yerkes Observatory. That part of the year is always unpropitious here, and it seemed as if everything combined, on this particular occasion, to

hide from us the growth of the comet and its approach to the earth. Forest fires in the northern part of the state produced a densely smoky sky which, even when the clouds were merciful to us and would have let us see the comet, cut off with a thick yellow veil all but a glimpse of the bright head. The sky, on every morning, was watched until strong daylight for a chance to photograph or observe the comet. Similarly every care was taken in the evenings to secure results as long as the comet was visible.

The transition from the morning to the evening skies by the passage of the comet between us and the sun on May 18 was coincident with a change in the weather conditions, and we were thus enabled to watch it in its recession from the earth and sun. After a long cloudy period the sky suddenly cleared at midnight, on May 17, and gave us a splendid opportunity that night and the night of May 18, at the most critical time, to observe the phenomenon of its nearest approach to us.

NAKED-EYE AND TELESCOPIC OBSERVATIONS

The following notes descriptive of the comet's appearance to the naked eye and with the telescope are given nearly in full in the hope that they may be of value at its future returns. Of course photography took care of the general features of the comet, and thus preserved an accurate record of its appearance to the sensitive plate. At the same time it is a noteworthy fact that the photograph usually gives but little information as to the naked-eye appearance of a comet. A careful description, therefore, of its appearance to the eye alone must have a special value, independent of that of the photographs, and supplemental to them. From a historical standpoint, for comparison with its appearance in times past, it must have a value beyond that of the photograph.

In the descriptions which follow I have, in some cases, gone rather extensively into the details of the naked-eye appearance of the comet. I feel that this is justifiable for the following reason. In looking up the published information concerning its appearance in 1835 to form some opinion as to how the comet would look at the present return, I was surprised at the meagerness of the records, and I determined to prepare as faithful an account as possible of its

appearance to the naked eye for the benefit of observers at future returns. I therefore made as accurate a record as I could of its appearance to the eye alone. These results were obtained while guiding on the comet in photographing it, and at other times when a few moments could be spared to examine it. The descriptions, after the comet came into the evening skies, were written down, from my dictation at the time, by my niece, Miss Mary Calvert, and therefore have the accuracy of the inspiration of the moment.

A pair of large, old-fashioned field-glasses were available in these observations, and were used to supplement the naked-eye views when necessary. These glasses were specially suited for the purpose, and far better, because of their large lenses, than the more modern field-glasses, which for such work are deficient in light, and generally are too powerful.

The Bruce photographic telescope is supplied with a 5-inch visual guiding telescope, with a field of about 20′. When photographing the comet, notes were kept concerning its appearance in this instrument.

THE NAKED-EYE AND TELESCOPIC NUCLEUS

One striking fact that was noticeable when the comet was bright in the evening sky, especially noticeable on or about May 26, was that it had a nucleus within a nucleus. To the naked eye the nucleus was stellar and as bright as δ *Leonis*, of magnitude 2.7. With field-glasses the nucleus was small, but of sensible diameter, and of a beautiful bluish-white color; it was surrounded by a much fainter hazy nebulosity, which ran out to form the tail—the view being rather an intensification of that with the naked eye. The naked-eye and field-glass "nucleus" was not the true nucleus. In the 5-inch guiding telescope a small planetary nucleus of the magnitude 8 or 9 was seen in a denser nebulosity. It was very well defined and very yellow. About this date, therefore, naked-eye and telescopic observations of the nucleus would refer to two entirely different things of exactly opposite colors. That which formed the nucleus to the naked eye was simply the small denser nebulosity about the real nucleus (see *Astronomische Nachrichten*, **185**, 234).

Following is a careful summary of the notes. The records belonging to the earlier part of the observations (containing also micrometer positions), when the comet was visible only in the telescope, and those similarly made in its later stages, have already been printed in the *Astronomical Journal*, **26**, 43, 62, 76, 1909–1910, and **27**, 147, 1912. The present notes all refer to the year 1910.

All the times recorded in this paper are Central Standard Time, or 6^h0^m slow of Greenwich Mean Time.

April 11, 15^h35^m. Examined the sky but could see no traces of the tail. There was a broad strip of haze in the east, but the horizon seemed clear for about 2° altitude. At 16^h30^m the comet was well seen in the 5-inch guiding telescope, but it did not look any brighter than when last seen in March. There appeared to be a dim hazy nucleus with some nebulosity. The brightest part of the comet was at least two magnitudes less than *B.D.*$+7°5121$ of magnitude 6.3. There was no trace of the tail; the sky was too bright and hazy to show it. The comet was visible in the guiding telescope until 16^h51^m, when it was lost. It is probable that it would have been faintly visible to the naked eye if the sky had been clear and dark.

April 13, 16^h5^m. It was quite easily seen in the 5-inch telescope as a brightish, ill-defined, nebulous star, with no trace of tail, and was lost in dawn at 16^h58^m. The sky at that time was more or less hazy. The comet was certainly brighter on this date than on April 11. Each previous morning, before the brightest dawn, the sky had been examined for any trace of the tail, but none could be seen.

April 16, from 15^h45^m to 16^h55^m. The comet was bright in the 5-inch telescope. When at a considerable altitude the nucleus was starlike, almost white, and of the sixth magnitude. It was not quite as bright as the star *B.D.*$+7°5121$ (magnitude 6.3) with which it was compared for brightness, though it was more conspicuous than the star. The tail could be traced for 15′ or 20′, but the comet was not visible to the naked eye. Judging, however, from its brightness in the 5-inch, it must have been close to naked-eye visibility. Clouds prevented any successful photographs.

April 19. When first seen at about 15^h20^m the comet was in a clear space close to the horizon. It was beautiful in the 5-inch, with a bright nucleus and a fine parabolic outline to the head, from

which the tail streamed out of the field of view. It was not visible with the naked eye, but the sky was too poor for one to have seen it.

April 20. The comet rose in dense haze, and was first visible in the 5-inch telescope at 15^h45^m, but was very dim. At 16^h20^m the nucleus was of the same brightness as the star *B.D.*$+7°5101$ (magnitude 7.0), but did not seem to be so intense in its light—it was more planetary and not quite starlike. The comet could not be seen with the naked eye at any time, the sky being too poor.

April 29. The comet was hidden by clouds until 15^h45^m, when it came out on a very bright sky, and could be seen with the naked eye for the first time. The nucleus was bright and was of magnitude 2 or 2.5. The tail was visible for a couple of degrees, but with field-glasses it could be traced for 4° or 5°. The comet remained visible to the naked eye until 16^h7^m, when it was lost, but it was visible in the 5-inch until 16^h30^m. To the naked eye it did not appear so bright as Daniel's comet in September of 1907 when in a similar position with respect to daylight.

May 2, 15^h40^m. The comet was seen for about one minute in a thin streak of clearer sky. The tail stretched out of the field of the 5-inch guiding telescope, but thick haze prevented its being seen with the naked eye.

May 3. The comet was beautiful to the naked eye, with a long tail streaming upward toward the right. The tail, however, was not bright. Before moonrise it could be traced for 17° or 18°. The head and nucleus were of about the second magnitude, and were estimated to be one magnitude brighter than γ *Pegasi*. Even after the moon rose the tail could be traced for nearly 15°. The following notes were made before the comet rose, the sky being examined carefully.

14^h17^m	No trace of tail.
14 22	No trace of tail.
14 29	No trace of tail.
14 34	No trace of tail. Sky good.

The comet was first seen at 14^h40^m. The smoke from the power-house was passing over it during most of the exposure, and must have cut the light down seriously.

May 4. The comet was beautiful. The tail stretched about one-half the distance to *θ Pegasi*, a length of 15°, and seemed a little shorter than on May 3. It became very gradually fainter toward the end, where it seemed to fade out as if that were really its end, and not so much as if it simply became too faint to be seen. The head, however, seemed brighter than on May 3, and was fully of the second magnitude. At about 15^h it was one magnitude brighter than *γ Pegasi*. At 16^h7^m the comet was still faintly visible to the naked eye, but one minute later it had disappeared. The smoke from the power-house was passing in front of the comet and partly hiding it during the observations, so that the exposures must have been cut down in effect at least one-half.

May 5. Dense, thick sky. No trace of the comet could be seen with the naked eye. It was very faintly visible in the 5-inch telescope.

May 6. The sky was very thick. The comet was fairly well seen with the naked eye when it rose, but hazy clouds at once covered it. At first the tail could be traced, even in the hazy sky, for a distance of 17° or 18°. The whole comet must have been brighter than at previous observations. It could still be seen faintly between the clouds with the naked eye at 14^h53^m.

May 8. The sky was very thick and was constantly being covered with heavy clouds. The comet was seen with the naked eye several times between the clouds. After 15^h10^m it seemed pretty bright with a long tail. The views were fragmentary through the breaks in the clouds.

May 9. The sky was very thick. At 15^h5^m the comet was seen very faintly with the naked eye. It must have been very bright to be seen at all under the conditions. The tail could be traced for 15°. The head was at least of the second magnitude. At 15^h48^m it was still faintly visible with the naked eye.

May 13. No trace of the comet because of dense haze and smoke all around the horizon. If the tail had been very long it would perhaps have been seen above the smoke bank.

May 14. The sky was very thick and bad. At 14^h40^m the tail could be traced slightly beyond *θ Pegasi*, a distance of about 53°, and passed about 2° or 3° to the right of and below that star. It must

have been 3° or 4° broad near the south side of the square of *Pegasus*. It was fairly noticeable when looked at with averted vision, but could not be traced anywhere near the head, which was invisible in the haze.

At 15^h40^m the comet was faint in the 5-inch telescope, while *Venus*, at the same altitude, was fairly well seen with the naked eye, but was very dull and red. At 16^h0^m the nucleus, which was yellow and starlike, with some coma, was quite noticeable in the 5-inch telescope, but neither the head nor any of the tail near it could be seen with the naked eye at any time, because of the smoky haze. Where the tail could be seen it was very straight and broad.

From the foregoing observations the head must have been many times less bright than *Venus*. The observations also show that the tail must have been about 50° in length.

May 17. After a stormy period the sky cleared brilliantly at midnight. As observations at this time are of the utmost importance in connection with the nearest approach of the comet to the earth, the notes will be given nearly in full.

13^h0^m. A narrow twilight (which later proved to be the tail) seemed to extend along the eastern horizon. This was more marked at 13^h5^m. "There is a diffused dawn effect near the east horizon about 4° or 5° high." At 13^h10^m this seemed either to have risen rather rapidly or to have become more pronounced. The sky was very clear, but the moon was still above the horizon. At 13^h25^m this "dawn" effect was as high as ϵ *Pegasi*. At 13^h30^m distinct traces of the tail were certainly visible a little south of the square of *Pegasus*, and reaching nearly to *Altair*. At 13^h35^m the axis of the tail would pass through θ *Pegasi*. It was perhaps 5° or 6° broad near θ and apparently rose to a point $10^\circ\pm$ southeast of *Altair*. At 13^h45^m θ *Pegasi* was in the axis of the tail. At 13^h55^m ζ *Pegasi* was on the north edge of the tail. At 14^h15^m θ *Pegasi* was close inside the south edge and slightly in the tail, while γ *Pegasi* was in the tail and perhaps one-half degree north of its middle or axis and θ *Aquilae* exactly on its north edge. At 14^h20^m the tail between ζ and γ *Pegasi* was perhaps brighter than any portion of the Milky Way. It seemed somewhat brighter in the middle and faded slightly toward the edges. It joined the Milky Way and could be

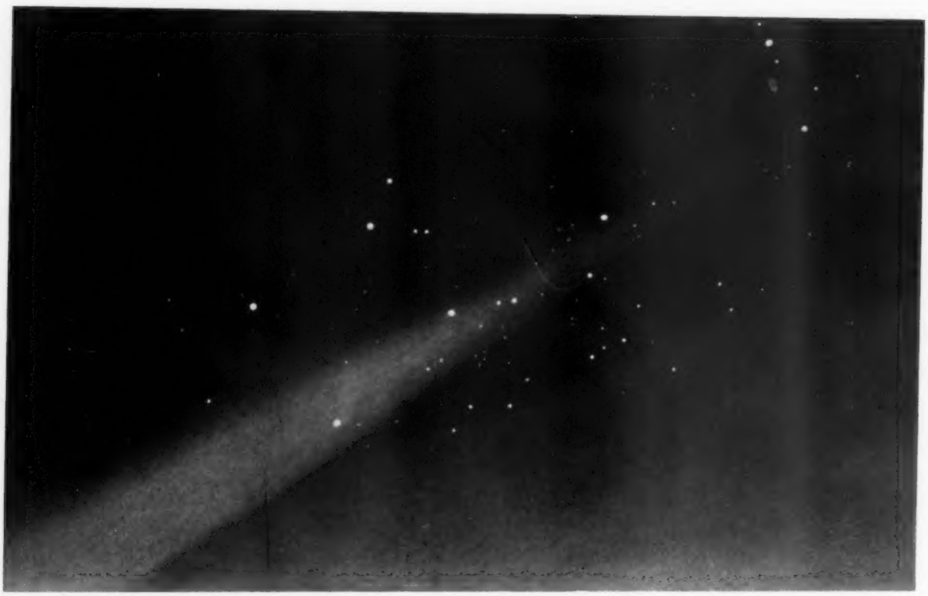
traced beyond θ *Aquilae*. At this time it appeared straight, but at about 13^h40^m it was thought to be slightly convex to the north. The width of the tail was a little greater than the distance from β to η *Pegasi* (5°). At 14^h25^m the tail, beyond ϵ *Pegasi*, was about one-fourth as bright or less than that part between ζ and γ *Pegasi*. The star 71 *Aquilae* (*B.D.* $-1°4016$) was in the middle or axis of the tail. At 14^h47^m α *Equulei* was just free of the north edge. At 15^h10^m the tail was faint from dawn and could be seen only by averted vision. At 15^h12^m it was still feebly visible near ζ and γ *Pegasi*, and could be traced as far as ϵ *Pegasi*. The sky was very clear. At this time γ *Pegasi* seemed to be still a little north of the axis. Miss Calvert watched it a little longer while I went to the 40-inch. She says that at 15^h18^m she could no longer see the tail, though she had seen it one or two minutes earlier.

The tail was only a little brighter toward the axis—it was very flat and did not diffuse much at its edges. Indeed it seemed to be nearly uniform in light with respect to its width, but it tapered very much toward the end, near which it would not be over three-fourths as wide as at a point near ζ *Pegasi*. This of course was an effect of perspective. Streamers or irregularities were carefully looked for but none was seen. The edges of the tail were smooth and uniform. At about 13^h15^m or 13^h30^m I could see the sky dark, below and above the tail, and there appeared to be a brightening along the southeast horizon, as if another portion of the tail were present. At 14^h45^m α *Equulei* was just free of the north edge of the tail. The head of the comet could not be seen when it rose, either with the 5-inch or the 40-inch telescope, because of the thick sky near the horizon. The observations show that the tail was at least 107° long on this date.

May 18. Beautifully clear all day, with a few flecks of clouds in the afternoon. A beautiful night with a three-fourths full moon. Every preparation had been made to photograph any phenomena that might develop during the night. There was a slight mistiness in the air. This was noticed only when, on hiding the moon, a feeble illumination was seen near it. At 8^h37^m and later, certain phenomena developed which are believed to have been auroral. The notes on these have been collected and are given later. At this

PLATE X

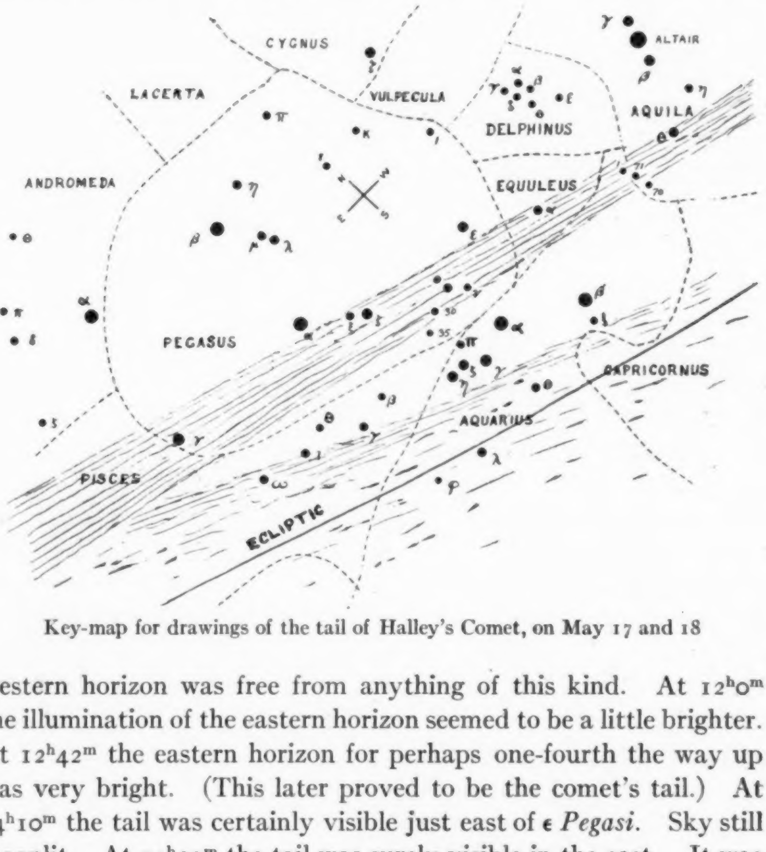
May 17, G.M.T. 21^{h}



May 18, G.M.T. 21^{h}

DRAWINGS OF TAILS OF HALLEY'S COMET

time the sky appeared unusually good. There was still a considerable twilight effect in the low northwest. At 8^h56^m some faint luminosity was visible under *Cassiopeia.* The sky had a feeble misty look everywhere, which was not due to ordinary haze, for apparently the sky was very clear. Up to 10^h35^m nothing out of the ordinary was noticed in the appearance of the sky, except the faint mistiness which had been visible since dark. At 10^h40^m the eastern horizon was bright with a diffused luminosity while the



Key-map for drawings of the tail of Halley's Comet, on May 17 and 18

western horizon was free from anything of this kind. At 12^h0^m the illumination of the eastern horizon seemed to be a little brighter. At 12^h42^m the eastern horizon for perhaps one-fourth the way up was very bright. (This later proved to be the comet's tail.) At 14^h10^m the tail was certainly visible just east of ϵ *Pegasi*. Sky still moonlit. At 14^h20^m the tail was surely visible in the east. It was quite noticeable at this time, even in the moonlight. It seemed to be a little north of its position of the previous morning (A.M. of May 18). At 14^h23^m γ *Pegasi* was just inside the south edge of the

tail, while α *Pegasi* was just inside it on the north edge. It lay between θ and ϵ *Pegasi*, nearer to θ. It was strongly visible (moon nearly down). At 14^h27^m the north edge of the tail diffused very gradually and reached halfway from γ *Pegasi* to α *Andromedae*. Both θ *Aquilae* and ζ *Pegasi* were in the axis of the tail. At 14^h42^m γ *Pegasi* was 3° inside of the south edge of the tail. The north edge diffused three-fourths of the way to α *Andromedae*, and was 10° wide near that star. At 14^h52^m the tail could be traced to the horizon, widening out toward the east horizon. At 15^h4^m the tail could still be seen (though it was dim) and the dark region in it. At 15^h10^m the tail and the dark space were still feebly seen, but they were badly dimmed by dawn.

The brightest portion of the tail near α and γ *Pegasi* was as bright as the Milky Way, but did not seem to be more than half as bright as on the previous morning. The tail could be traced to the Milky Way beyond θ *Aquilae*, where it became faint and somewhat tapered. It could not be traced across the Milky Way. Several times the impression was given—I was almost sure of it—that the brightest part of the tail fluctuated in brightness as if its light were unsteady. The illumination below the dark space in the tail, though feeble, I think was real. It apparently extended to the southeast horizon as if it passed below the horizon, and there seems no question that it was a separate part of the tail, and that the dark space was a rift that separated the tail into two parts. There was no evidence of any streamer north of the bright tail. The south edge was rather definite, though softly blended, but the north side was very diffused. At times it seemed to diffuse beyond α *Andromedae*. The light of the tail was very similar to that which forms the *Gegenschein*, or like light reflected from dust particles; that is, it did not have a nebulous appearance. The bright northern part of the tail could be said to be roughly cone shaped, with its base along the horizon, and tapering out and becoming faint toward θ *Aquilae*. As dawn approached, say a little after 15^h, the whole sky seemed to assume a feeble glow that did not appear to be entirely due to dawn. The observations, located on a celestial globe, make the length of the tail at least 120°.

On both nights (May 17 and 18) there was no decided light north

of the tail, that is, all the sky above the tail was apparently pure and free from unusual illumination, with the exception of the slight mistiness mentioned in the preceding paragraph. The illumination below the brighter part of the tail was decided. It was soft and seemed to reach to and beyond the southeast horizon.

Although the slight aurora which developed on May 18 certainly had nothing to do with the proximity of the comet, it seems best to give it here as a part of the record for that night for comparison with observations that were doubtless made elsewhere and for other reasons. I have therefore collected all the phenomena that certainly seemed to belong to the aurora in the notes that follow.

At 8^h37^m there seemed to be some horizontal streaks of diffused light in the north above *Cassiopeia*. They had disappeared five minutes later. At this time considerable twilight effect still remained in the northwest. At 8^h56^m some feeble luminosity was visible under *Cassiopeia*. At 9^h2^m apparently a slight aurora was visible to the right of and below *Cassiopeia*. At 9^h14^m a faint luminous band was visible halfway from the horizon to the stars of *Cassiopeia*. This seemed to be an auroral effect. At 9^h20^m an active aurora with streamers flashed up very suddenly. By 9^h28^m it had become a uniform glow extending almost as high as *Cassiopeia*. There were feeble attempts at activity at 9^h40^m consisting of a great number of short streamers. At 10^h4^m the altitude of the bright part of the arch (which was fairly strong) was exactly halfway from the horizon to α *Cassiopeiae*. At 10^h12^m streamers were ascending to the left of the summit of the arch and moving to the left. At 10^h16^m the arch was rather strong but indefinite. At 10^h18^m the aurora was again active (but not bright), with diffused streamers. At 10^h28^m there was no definite arch, but a diffused general illumination was present reaching nearly as high as *Cassiopeia*. At 10^h32^m still diffused, with no definite arch, and one streamer moving west. The brightest part of the illumination extended halfway to *Cassiopeia*. By 10^h40^m the aurora had almost faded out—apparently dead. At 11^h10^m a very slight auroral glow. At 11^h24^m there seemed to be no aurora. At 11^h32^m, no aurora. At 13^h52^m the aurora started up again with a very low feeble arch. At 14^h14^m the altitude of the arch was $3°$ or $4°$.

There are no other notes on the aurora, and I assume that it finally disappeared about this time.

Notes were also made of the appearance of any meteors, but they are not given here because but very few were seen on May 18 and none was noted on May 17. They seemed to have no connection with the comet.

May 19. Cloudy at night. No observations of the comet possible.

May 20. At 7^h50^m to 8^h30^m with the naked eye the head was one-half degree in diameter. The head and nucleus were of about the second magnitude, and resembled a yellow nebulous star. There seemed to be a faint diffused tail. The sky was muggy, and the comet was in clouds most of the time.

In the 5-inch the nucleus at first was very stellar and very yellow, with some of the hazy yellow light about it, but no tail was seen with certainty.

The sky was examined repeatedly as late as 11^h, but no trace of the comet's tail was anywhere visible. The moon was nearly full.

At 14^h the sky was hazy in the west. At about 14^h30^m a hazy luminous streak $4°$–$5°$ broad extended from θ *Aquilae* to the east —fainter toward θ *Aquilae*—through α *Pegasi*. This resembled the comet's tail, but was doubtless a strip of haze. I looked at it several times, taking it for a strip of haze, but it did not seem to move. There were masses of moving haze overhead toward the north. To all appearances it looked like the comet's tail of the mornings of May 18 and 19. I cannot be certain that this was not haze, but it was a singular coincidence of position, appearance, etc., if it was. It was visible for fully 15 or 20 minutes. Then the sky got worse with the haze and moonlight and it disappeared. At the same time there seemed to be a similar strip in the low south which stretched from the Milk Dipper in *Sagittarius* to *Antares*. This was $3°$ to $4°$ wide. I think these must have been merely strips of haze, and had nothing to do with the comet, but they are given here as a matter of record. It may be well to note, however, in this connection that the tail—both branches—was still visible in the morning sky in South Africa on this date, at 15^h30^m G.M.T., or

only five hours earlier than the supposed observation recorded above (see *Circulars 3* and *11*, Transvaal Observatory). Both tails were also seen by Perrine at Cordoba on the 20th at the same moment as my observation (see *Astronomical Journal*, **26**, 145).

May 21 and 22. Cloudy.

May 23, 8^h40^m. Sky cloudy, but the comet shone for a few minutes through a break in the clouds. It was very bright—perhaps brighter than the first magnitude. To the naked eye the nucleus and coma appeared like a nebulous star. There was some faint tail. During the exposures the sky was white with a full moon and haze, patches of which frequently covered the comet. The total eclipse of the moon on that night unfortunately came too late to aid in the observations of the comet.

May 24. At 7^h55^m the comet was quite bright to the naked eye, with traces of the tail. It was bluish white and a striking object. The head was large and hazy and about $15'$ or $20'$ in diameter. The nucleus resembled a first-magnitude star in haze. The tail was $25°$ long. For $5°$ or $6°$ it was noticeable and then became rapidly fainter. At 8^h35^m it was quite noticeable for $10°$ or $15°$. At $10°$ from the head the tail was about $2°$ wide. The light of the comet was still bluish white. At 8^h50^m the tail could be traced with the naked eye for $29°$. At 9^h10^m the sky was very bright with moonlight and did not seem to be very clear, but the tail was still noticeable to the naked eye. Ten minutes later the tail was very feeble for want of contrast and at 10^h0^m it was scarcely visible to the naked eye, but the head was still very bright like a hazy star. At 10^h30^m the comet was still visible, very low and dim.

In the 5-inch telescope the nucleus was sharply defined, not a point, but more like a small bright planet with coma. At 8^h45^m it was nearly white. It also appeared white to the naked eye. In the last part of the exposure the nucleus was about five times greater in diameter than when the exposure began, and more ill defined—it seemed to swell in size.

[The last part of the foregoing paragraph is in accord with the observations of Professor A. E. Douglas at Tucson, Arizona, who later, on this same date, saw the nucleus double. See *Harvard Observatory Bulletin*, No. 412.]

May 25. At 8^h30^m the tail could be traced for 21°. The head was decidedly less intense than ϵ *Hydrae* south of it. The sky was quite good. At 8^h50^m the tail could be traced for eight-tenths the distance between the head and *Jupiter*, or a length of 43°. At 8^h55^m the comet was seen on a fairly dark sky, only a little twilight effect remaining. It was very beautiful, though the head did not seem relatively so bright as on other nights. The tail, for about 20°, was pretty bright, and increased very much in width. At 9^h0^m it seemed to extend in a very diffused manner nearly to the same right ascension as that of *Jupiter*, a distance of 54°. Prolonged, its axis would pass about 8° south of *Jupiter*. The tail was very diffused at its end and seemed to extend northward nearly to *Jupiter*. At 9^h10^m the central brightness of the head was almost bluish white in the field-glasses. At 9^h30^m the sky had begun to whiten with moonlight, but the comet was still in good relief, the moon being behind clouds in the east. At 9^h35^m the tail could be faintly traced several degrees beyond 87 *Leonis*= *B.D.*$-2°3360$ (magnitude 5.0), which at this time was in the axis of the tail, or for about 43°. At 9^h55^m the tail was very dim on the bright moonlit sky, but was still faintly visible for 10° or more.

At 8^h30^m with the 5-inch telescope the nucleus was very small, like a ninth- or tenth-magnitude star. The coma was very large and fairly bright. Before the wires were illuminated the nucleus did not appear double, nor were there any other nebulosities in the field of view. It was very dim and hazy. At 8^h50^m the nucleus was of the same brightness as *B.D.*$+7°2055$ (magnitude 8.4). At 9^h15^m the nucleus was a little brighter and hazy. At 9^h35^m it was fairly stellar and a little brighter—brighter than any of the stars in the field of view: (*B.D.*$+7°2048$ [magnitude 9.0], $+7°2052$ [magnitude 9.2], $+7°2055$ [magnitude 8.4]). At 9^h50^m the coma was very dense and extended perhaps 5′ all around the nucleus, which was very small and dim.

May 26, 7^h55^m. The comet was visible to the naked eye as a faint hazy star. At 8^h5^m it was quite noticeable, with perhaps faint traces of tail. At 8^h22^m the tail was showing faintly to the naked eye. The comet seemed less bright than on the previous night. At 8^h30^m, with field-glasses, there seemed to be a central

nucleus, fairly well defined, large, and bluish white, surrounded by much fainter hazy nebulosity which extended from it to form the tail. It had the same appearance to the naked eye. This, however, was not the true nucleus, which was very small and seen only in the telescope. The tail was visible for 10° or 15°. At 8^h40^m the tail was very noticeable for about 15°, but it faded very rapidly toward the end. To the eye the nucleus was bright—of the second magnitude. At 8^h45^m the sky was very good and the tail was very noticeable. It could readily be traced to 87 *Leonis*. With field-glasses the nucleus was an intense bluish white. The whole head seemed to be of a bluish-white color. At 8^h50^m the tail was conspicuous halfway to 87 *Leonis*, after which it became diffused and faint. It seemed a little brighter in the middle near the head. No streamers were seen. The sky was fairly dark and the comet a conspicuous and strikingly beautiful object. But the nucleus was very much inferior to *Regulus*. At 8^h55^m the tail was conspicuous as far as 87 *Leonis*, and, though rather faint near that star, it could be traced feebly 10° beyond it. To the naked eye the nucleus was *very* much brighter than the rest of the head. At 9^h0^m the tail could be very feebly traced beyond *Jupiter*. The axis would pass 6° or 7° south of the planet. At 9^h10^m the nucleus was about as bright as δ *Leonis* (magnitude 2.6). The comet was a very striking object to the naked eye, with the tail, which seemed to be straight, reaching as far as 87 *Leonis*, where it became faint. At 9^h15^m, by hiding *Jupiter*, the tail could be feebly traced to α *Virginis*, or a length of about 65°. The nucleus was perhaps one-half a magnitude less bright than γ *Leonis* of magnitude 2.6. At 9^h20^m the sky was still good. The tail for 15° from the head was everywhere brighter than *Praesepe*. Within 5° or 10° of the head it was 4 or 5 times as bright as *Praesepe*. At 9^h45^m the comet was seen on a fine dark sky and was very conspicuous. In the field-glasses the tail widened out very much. The nucleus was large and bluish white and was surrounded for a short distance by a hazy glow of the same color. There seemed to be no structure in the tail. At 9^h50^m the moon was whitening the eastern sky, but the tail was still noticeable as far as 87 *Leonis*, where it became faint. It gradually widened out, with the south side perhaps a little the

brighter. To the naked eye the nucleus was rather dull. In the field-glasses it was still bluish white and hazy, like a star shining through a bluish-white mist. There did not seem to be any evidence of streamers, either with the naked eye or with field-glasses. At 10^h20^m the sky was very bright with moonlight, but the comet's tail was still noticeable (though not very strong) for some $10°$, and could be traced faintly as far as 87 *Leonis*. At 10^h30^m the tail could still be traced feebly for $10°$ or more from the head. At 10^h50^m the comet was very near the horizon and disappearing in some tree tops, and nothing could be seen of the tail with the naked eye.

At 7^h55^m, with the 5-inch telescope the nucleus was small and planetary, and with the coma was very yellow. The nucleus was larger and perhaps slightly brighter than *B.D.*$+6°2129$ (magnitude 8.0), which was in the field. At 8^h40^m, though less intense, it was brighter, and more yellow than the star. It seemed to be very much brighter than on May 25. The nucleus was estimated to be decidedly brighter than the star at 9^h5^m, and at 9^h10^m it was stated that it must have brightened since the exposure began. It was ill defined and perhaps $5'' \pm$ in diameter. At 9^h35^m the nucleus was very much brighter. It seemed to have increased greatly in brightness but was very ill defined. At 10^h5^m it was decidedly more yellow and perhaps a little brighter than the same star, though its light was not so intense. It was very hazy and much larger than the star. At 10^h50^m the comet and nucleus were both very faint.

May 27, 8^h0^m. To the naked eye the comet resembled a small dim cloud, in which the nucleus was small and faint. The sky was smoky, and had been so almost all the late afternoon. At 8^h15^m the comet was dull to the naked eye, like a dull nebula some $10'$ or $15'$ in diameter. One could not be sure of seeing any tail at this time. At 8^h25^m the tail could be feebly seen for $4°$ or $5°$. At 8^h35^m it was only feebly visible for perhaps $5°$ or $6°$, but it could be seen fairly distinctly. The sky was very poor with some twilight illumination. At 8^h38^m the tail could be seen rather dimly for about $10°$—sky still luminous. In the field-glasses the condensation or nucleus was of a bluish-white color. The rest of the head and

tail were whitish. At 8^h42^m the tail could be traced faintly to 87 *Leonis* (about 33°), and near the head for 6° or 8° it was quite noticeable. The comet did not seem as bright as on May 26, but the sky was poor and whitish. The nucleus was about midway in brightness between γ (magnitude 2.6) and ζ *Leonis* (magnitude 5.1), or about magnitude 3.8. At 9^h0^m the tail seemed to be brighter on the south side and could be traced quite distinctly to 87 *Leonis* (which star was apparently in the axis, or perhaps a little south of it), after which it became faint. For one-half that distance it was conspicuous. The sky was fairly dark but it was not pure. With the field-glasses the tail near the head was feebly brighter in the middle. At 9^h10^m the condition of the sky, though not pure, was fair. Possibly the tail was slightly curved, with the convex side south. It was quite noticeable as far as 87 *Leonis*. At 9^h15^m there did not seem to be any structure in the tail as seen with the field-glasses. To the naked eye the comet was a conspicuous object. The tail near the head was very much brighter than *Praesepe*, but it faded off rapidly near 87 *Leonis*. By hiding *Jupiter* it could be traced to a point halfway between *Jupiter* and α *Virginis*, or for a distance of 53° or 54°. It seemed certainly to be curved when the whole tail was considered, with the convex side toward the south. Near *Jupiter* it was perhaps 3° in width and faint. At 9^h30^m, with the field-glasses, what appeared to be the nucleus was of sensible diameter and hazy and was very strongly conspicuous. At 9^h35^m there seemed to be a diffusion from that part of the tail near *Jupiter*, extending to the north as high as the planet (α 12^h19^m, $\delta - 0°31'$). The star 87 *Leonis* was perhaps a little south of the middle of the tail. The nucleus was in pretty strong contrast to the tail near the head. At 9^h40^m the tail, from 87 *Leonis* to the end, became exceedingly faint and diffused. At 9^h45^m the comet was still a conspicuous object, with the tail extending to 87 *Leonis*. The nucleus resembled a dull hazy star of the third magnitude. The sky was fair, though not specially pure. At 10^h20^m the comet's head was getting down into the haze near the horizon, but was still strongly conspicuous. By 10^h30^m the head was becoming dim to the eye. The tail was still noticeable and could be traced readily to 87 *Leonis*. At 10^h40^m the head

was very dim. The tail also was very dim, but could still be traced to 87 *Leonis*. The sky up high and overhead was very clear, but near the horizon there was a good deal of smoky haze.

At 8^h5^m the nucleus, in the 5-inch telescope, was faint and ill defined, with some haze about it. It was very much fainter (perhaps two magnitudes) than the star *B.D.*$+5°2171$ of magnitude 8.1, which was in the field with it. At 8^h45^m the nucleus was very dim, and was very feebly contrasted with the nebulosity. At 8^h50^m what was so conspicuous as a nucleus to the naked eye could not have been the true nucleus, for in the 5-inch the nucleus was very small and faint and was apparently only a condensation in the coma. It must, therefore, have been the brighter part of the coma which formed a nucleus to the naked eye. At 9^h10^m it seemed to have grown dimmer. It was rather difficult to guide on and was very small and ill defined. At 9^h35^m the nucleus and coma appeared very much like the nucleus and close nebulosity of the Great Nebula of *Andromeda* when seen in an ordinary telescope, and showed about the same amount of contrast, the bright part of the coma being about $1'$–$2'$ in diameter and very diffused. At 9^h50^m the true nucleus was very small and dim, and was several times fainter than the star *B.D.*$+5°2171$. It was surrounded by a dense nebulosity $1'$ or more in diameter. This nebulosity must have been what appeared to the naked eye as the nucleus. At 10^h5^m the nucleus was difficult to guide on. The glow about it was very strong and there was no contrast. It was simply a central condensation of the coma. At 10^h15^m the nucleus was just discernible, being all but lost in the coma. At 10^h35^m it was no longer visible to guide on. I do not think its faintness was due entirely to the condition of the sky.

May 29. At 10^h5^m the tail was conspicuous as far as 87 *Leonis*, a distance of 27°, its axis passing about one-half degree north of that star. It could be feebly traced beyond the line between *Jupiter* and *Spica*, or about 52°, but only feebly. It was noticeably curved—convex to the south. The southern side, from the head to 87 *Leonis*, was a little the brighter and more definite. The nucleus was about as bright as η or θ *Leonis*. The tail seemed to diffuse to the north to *Jupiter*, and perhaps beyond. The southern

PLATE XI

North

10-inch Lens. May 29, $15^h 39^m$, G.M.T. Exposure, $1^h 57^m$
Scale: 1 cm = $0^\circ.55$

edge would about bisect the line between *Spica* and *Jupiter*. The comet was decidedly less bright than on May 27.

In the 40-inch the nucleus was not yellow, but was pale in color. The measured diameter north and south from one setting was $2''.6$.

May 30. At 8^h5^m the comet could be feebly seen with the naked eye like a faint nebula. At 8^h20^m it was not as bright nor as noticeable as the star π *Leonis*. There was no tail visible at this time. The sky was very clear. At 8^h30^m with the field-glasses the tail could be traced for a couple of degrees. The head was quite bright and seemed to be a nebulous mass without any special nucleus. There was perhaps a faint suggestion of a tail for a couple of degrees, but very faint. The head was about as bright as π *Leonis* (magnitude 4.9), perhaps a little brighter, but more noticeable than that star. At 8^h40^m the tail could be traced as far as 87 *Leonis*, a distance of 25°, but was faint toward its end. The sky was still bright with twilight. At 8^h50^m the comet was quite conspicuous. The tail was noticeable as far as 87 *Leonis* and was seen faintly 7° or 8° beyond that star (which seemed to be nearly in the axis of the tail). For about 15° it was conspicuous. The head was of about the third magnitude, and with the field-glasses resembled a bright hazy nebulosity. At 9^h15^m the tail seemed decidedly curved between the head and 87 *Leonis*, with the convex side to the south. At about 5° from the head it was of about the same brightness as *Praesepe*. Nearer the head it was brighter. With the field-glasses the condensation in the head looked like a large diffused nucleus, bluish white in color, surrounded by a fainter nebulosity which extended back to form the tail. The central brightness was very strong as compared with the rest of the head. At 9^h25^m the comet, to the naked eye, was very dull as compared with its appearance a few nights earlier, but it was still conspicuous. The tail, where it passed below *Jupiter*, had the same appearance of diffusing and spreading out toward that planet previously noted, but *Jupiter* was too bright to make this certain. The sky was very transparent, especially in high altitudes. At 9^h40^m the comet was a striking object. The tail was conspicuous as far as 87 *Leonis*, after which it became faint, but by hiding

Jupiter it could be feebly traced as far as the line between *Jupiter* and *Spica*, a length of 47°. With the field-glasses no structures or irregularities could be seen in the tail, which diffused very softly toward the edges and was not especially brighter in the middle. No streamers were visible either with the naked eye or with the field-glasses. At this time it was still a conspicuous object. At 10^h30^m, the comet, though low, was still conspicuous. The sky seemed to be very good in its direction. The head was quite bright, and the tail could be readily traced to 87 *Leonis*. At 10^h45^m the head was quite bright, like a second- or third-magnitude star, but the tail was lost in clouds.

During the exposures on the comet there seemed to be a denser part some 10° back from the head, as if the tail sagged a little south at that point.

At 8^h5^m, though the head was distinct to the naked eye, it was very small and faint in the 5-inch guiding telescope. At 8^h35^m the nucleus was very small and starlike and shone in the middle of a dense nebulosity about $0'.5$ in diameter. It was one magnitude brighter than the star *B.D.*+3°2273 of magnitude 9.2. At 9^h10^m the nucleus was very small and starlike in a very dense nebulosity which diffused gradually for $1' \pm$. It was very much brighter than *B.D.*+3°2273—about $1\frac{1}{2}$ magnitudes brighter. The sky was very clear. At 9^h45^m the nucleus was very small and stellar with some haze close about it. It was not very much brighter than *B.D.*+3°2273. At 10^h17^m the nucleus was almost lost in the strong condensation about it.

May 31. The comet was first seen with the naked eye at 8^h17^m. The sky was covered more or less with hazy clouds, but at about 10^h45^m, when seen for a few minutes below the clouds, it was conspicuous.

In the 40-inch telescope at 8^h42^m the measured diameter of the nucleus, north and south, was $4''.9$.

June 1. At 8^h10^m the comet was faintly visible to the naked eye. At 8^h40^m only faint traces of the tail could be seen. The sky at this time was covered with hazy clouds from the northwest. At 9^h0^m, in spite of the condition of the sky, the tail could be traced to 87 *Leonis*, a distance of 22°. The comet was covered with hazy

clouds nearly all the time. There were long strips of these clouds moving southwardly, which were, most of the time, only a few degrees wide, and if they had been displaced 4° or 5° the comet would have been seen on a good sky throughout the observations. Once in a while it came out for a few minutes only to be covered again. The rest of the sky was good. After 10^h it got on to a better sky and there was very little interference from clouds, but the sky was not good in the direction of the comet. The tail could be traced for several degrees beyond 87 *Leonis*, or perhaps for about 25°, and was noticeable as far as that star. The head, which seemed brighter on this date, was about midway in brightness between γ and η *Leonis*, or $3^m.1$. At 11^h0^m, though the comet was very low and dim, the tail, in moments of freedom from clouds, could be seen up to 87 *Leonis* fairly well, and could be traced some degrees farther. Its axis passed slightly north of that star.

At 8^h10^m in the 5-inch the nucleus was very faint and small —just visible—in a strong condensation. At 8^h58^m it could no longer be seen to guide on.

June 5, 9^h7^m. To the naked eye the head was about one-half magnitude brighter than the star 15 *Sextantis*=*B.D.*$+0^\circ2615$ (magnitude 4.1) and more conspicuous than that star. The tail, which seemed to be straight, could quite readily be traced to 87 *Leonis* and perhaps a few degrees beyond, but it was dim. It was visible in a diffused manner to the star χ *Virginis*, or about 33°. To the naked eye a faint nucleus was doubtfully visible. The head was about as bright as 87 *Leonis*, and not much more noticeable than that star.

The comet was first seen in the finder of the 40-inch at 8^h9^m. With the 40-inch telescope itself at 8^h20^m, the nucleus was very small, $2''$ or $3''$ in diameter, and surrounded by a dense nebulosity. At 9^h0^m it was a very small point in dense hazy light which was placed in a very strong nebulosity which faded away rapidly, and was perhaps $3'$ or $4'$ in diameter. The minute nucleus was about three magnitudes less than the comparison star (estimated magnitude $9\pm$; see *Astronomical Journal*, **27**, 149, 1912), but the general brightness of the head would be about $2\frac{1}{2}$ magnitudes less than the star.

In the 5-inch telescope no nucleus was visible. There was only a strong condensation which was rather hard to guide on.

June 6. At about 8^h27^m the comet became visible to the naked eye as a faint hazy spot. At 8^h47^m the tail was not yet visible. The head was not quite as noticeable as the star *ρ Leonis* (magnitude 3.8). At 8^h50^m the tail could be traced for a distance of 5°. At 8^h57^m it could be seen faintly to 87 *Leonis*, a distance of 18°. At 9^h10^m it was noticeable as far as 87 *Leonis*, which was on its upper (north) edge, and could be seen feebly several degrees beyond. Sky good. At 9^h57^m the tail could be traced to *χ Virginis*, or for 32°. Though very faint, it was noticeable as far as 87 *Leonis*. The comet had faded sadly, however, since June 1, and though a noticeable object, was only the ghost of its former self.

At 8^h25^m it was quite conspicuous in the 5-inch telescope, with a bright starlike nucleus, which was about one-half magnitude less bright than the star *B.D.*$+0^\circ2641$ (magnitude 8.0). At 8^h35^m the nucleus was beautifully starlike, and imbedded in a very strong condensation that faded rapidly and was itself nebulous. When best seen at 9^h42^m the nucleus was about one magnitude less than *B.D.*$+0^\circ2641$, or about the ninth magnitude.

June 7. At 9^h40^m the tail was faint, but could be traced to 87 *Leonis* (which star was in the north edge of the tail), a distance of 17°. The entire comet was fainter than on June 6. The head was of about the same brightness as the star *ρ Leonis*. It was relatively fainter with respect to the tail than at previous observations. The sky was poor and the Milky Way dull. There were no clouds, however.

The comet was first visible in the finder of the 40-inch at 8^h8^m. In the 40-inch telescope itself the nucleus, which was in a very strong condensation, was very ill defined and blurred.

June 9. At 10^h35^m the sky was murky and broken with clouds. The comet was only fairly visible to the naked eye. At best the tail could be very faintly traced to 87 *Leonis*, or for 15°. In the latter part of the observations the sky was good everywhere else but in the region of the comet, which was covered with misty clouds.

PLATE XII

North

10-inch Lens. June 6, $15^h 49^m$ G.M.T. Exposure 120^m

Scale: 1 cm = $0^\circ.44$

GREATEST VISIBLE LENGTH OF THE TAIL

For the convenience of those interested in the matter, Table II contains the greatest lengths of the tail as seen with the naked eye during these observations.

TABLE II

Date	Length of Tail	
May 3	17°–18°	
4	15	
6	17 –18	On hazy sky
9	15	On very bad sky
14	53	
17	107	
18	120 or more	
24	29	
25	54	
26	65	
27	53	
29	52	
30	47	On poor sky
June 1	25	
5	33	
6	32	
7	17	
9	15	

A list of the photographs obtained with the various lenses of the Bruce photographic telescope is given in the catalogue of the report of the Comet Committee of the Astronomical and Astrophysical Society of America.

YERKES OBSERVATORY
WILLIAMS BAY, WIS.
March 7, 1914

ELEMENTS OF THE ECLIPSING VARIABLE STARS *Z DRACONIS* AND *RT PERSEI*

By HENRY NORRIS RUSSELL AND HARLOW SHAPLEY

The very accurate light-curves determined by Dugan for *Z Draconis* and *RT Persei* give an opportunity for carrying the solutions for the orbital elements to a much higher degree of precision than is usually practicable. The following discussion will give the application of the method of least-squares to the derivation of definitive elements from photometric work, and will treat of an annular eclipse when the star's disk is supposed to be darkened at the limb.[1] The observations afford conclusive evidence that in both cases the stars are really darkened at the limb, and in the case of *RT Persei* make possible the determination of the eccentricity and the longitude of periastron from the photometric data alone.

I. The star *Z Draconis* (*B.D.*$+73°533$, $11^h39^m.8$, $+72°49'$) was found to be an *Algol* variable of large range by Mme. Ceraski in 1903.[2] The present discussion is based upon the photometric observations by Dugan, published in No. 2 of the *Contributions from the Princeton University Observatory.* These number 1149, each representing the mean of 16 settings, and cover the entire period, somewhat more thickly during the times of eclipse than elsewhere. The period, $1^d8^h34^m40^s.95$, and the epoch of principal minimum are taken from Dugan's discussion (*op. cit.*, p. 16), and normals have been formed from the data given in his table of observations arranged in order of phase (pp. 19–39).

The resulting normals for that part of the light-curve which lies outside the principal minimum are given in Table 1. The first column gives the number of observations combined into a normal—18 during the secondary minimum, and usually 36 outside eclipse—the second the mean phase, counted from principal

[1] For a summary of the formulae and notation, and references to the original discussions and tables, see *Astrophysical Journal*, **36**, 404, 1912. All references in the text to tables with Roman numbers are to these papers.

[2] *Astronomische Nachrichten*, **161**, 159, 1903.

minimum, and the third the mean observed light-intensity, derived from the values given by Dugan. The unit of intensity corresponds to a stellar magnitude fainter by 1.477 than his comparison star, *B.D.*+72°545—that is, according to the data of p. 4 of his memoir, to the magnitude 10.46 on the Harvard scale. The fourth column gives the residuals from the definitive solution described below, and the remaining columns the coefficients of the equations of condition from which this solution was derived.

TABLE 1

No. Obs.	Phase	Observed Intensity	O−C	Coefficients of			
				δb	δc	δd	e
36......	+ 3^h26^m	0.973	−0.008	−0.79	−0.63	....	....
36......	4 48	0.982	−0.006	−0.60	−0.36	....	....
42......	5 38	1.002	+0.010	−0.46	−0.21	....	....
36......	6 20	0.981	−0.014	−0.34	−0.12	....	....
36......	7 01	1.001	+0.003	−0.20	−0.04	....	....
36......	7 43	0.992	−0.010	−0.10	−0.01	....	....
36......	8 35	1.010	+0.004	+0.09	−0.01	....	....
42......	9 25	1.011	+0.002	+0.25	−0.06	....	....
36......	10 08	1.018	+0.007	+0.38	−0.15	....	....
36......	11 08	1.009	−0.003	+0.55	−0.30	....	....
42......	+12 50	1.006	−0.008	+0.79	−0.62	....	....
18......	+14 04	1.007	−0.005	+0.92	−0.85	−0.04	+0.04
18......	15 06	1.008	+0.022	+0.98	−0.97	−0.40	+0.12
18......	15 38	0.941	−0.013	+1.00	−1.00	−0.85	+0.12
18......	16 36	0.945	+0.002	+1.00	−1.00	−0.99	−0.02
18......	17 11	0.959	0.000	+0.99	−0.98	−0.68	−0.13
18......	17 48	0.999	+0.016	+0.97	−0.93	−0.30	−0.12
18......	18 27	0.989	−0.014	+0.91	−0.83	−0.16	−0.07
24......	+19 05	1.002	−0.011	+0.85	−0.72	−0.03	−0.02
36......	−12 00	1.017	+0.003	+0.68	−0.46	....	....
36......	−10 30	1.021	+0.010	+0.44	−0.19	....	....
36......	− 8 51	0.998	−0.011	+0.14	−0.02	....	....
36......	− 7 13	0.991	−0.008	−0.18	−0.03	....	....
36......	− 5 45	0.992	0.000	−0.44	−0.19	....	....
36......	− 4 36	1.009	+0.023	−0.63	−0.40	....	....
27......	− 3 27	0.982	+0.001	−0.79	−0.63	....	....

The characteristics of this portion of the light-curve have been clearly interpreted by Dugan (pp. 41–42). There is a shallow but unmistakable secondary minimum, which follows the primary at an interval slightly greater than half the period. Outside eclipse the intensity rises toward secondary minimum, showing that the companion is brighter on the side which receives the radiation of the principal star than on the other; and the light-curve is

slightly convex upward, indicating a small ellipticity of the stars. Values of the constants involved in these relations are given by Dugan; but these have not been derived by a least-squares solution, and the number and accuracy of the observations justify such treatment.

If θ is the orbital longitude of the bright star, measured from the principal conjunction, and l the observed intensity, we should have:

$$l=a-b\cos\theta-c\cos^2\theta-nd\ ,$$

where $2b$ represents the difference in light-emission of the two sides of the companion, $1-c$ is the ratio of the minor to the major axes of the prolate spheroidal stars, d is the depth of secondary minimum, and n the loss of light at the corresponding phase during principal minimum, in terms of the loss at mid-eclipse. (This assumes that the intensity-curves of the two minima are of similar form, and differ only in depth —which will be fully justified by the elements given later.)

From a preliminary solution, the approximate values were found $a=1.004$, $b=0.021$, $c=0.010$, $d=0.072$, the middle of the secondary minimum being at phase 13^h30^m. In the equations of condition summarized in Table 1, δa, δb, δc, δd represent corrections to be added to the foregoing values, and $200\,e$ is the correction in minutes to the assumed time of secondary conjunction. The coefficients of this last quantity were read from a plotted curve. All the normals outside eclipse were given weight 1, and those during the secondary minimum weight $\frac{1}{2}$. The resulting normal equations are:

$$\begin{aligned}
+22.00\delta a+2.60\delta b-8.08\delta c-1.72\delta d-0.04\ e&=-0.0065\\
+\ 2.60\delta a+8.08\delta b-3.00\delta c-1.70\delta d-0.04\ e&=-0.0025\\
-\ 8.08\delta a-3.00\delta b+5.30\delta c+1.68\delta d+0.04\ e&=-0.0006\\
-\ 1.72\delta a-1.70\delta b+1.68\delta c+1.22\delta d+0.00\ e&=-0.0012\\
-\ 0.04\delta a-0.04\delta b+0.04\delta c+0.00\delta d+0.034e&=+0.0001\ ,
\end{aligned}$$

whence we find:

	Least-Squares	Dugan
$\delta a=-0.0008$,	$a=1.003\pm0.0023$	1.006
$\delta b=-0.0008$,	$b=0.020\pm0.0029$	0.016
$\delta c=-0.0014$,	$c=0.009\pm0.006$	0.016
$\delta d=-0.0013$,	$d=0.071\pm0.009$	0.059
$e=+0.002\pm0.037$		
Secondary conjunction	$13^h30^m\pm7^m$	13^h30^m

The corrections are all insensible, which is not surprising, as the preliminary solution was made on least-squares principles. The quantities O−C given in Table 1 are therefore at the same time the absolute terms of the equations of condition and the final residuals. The probable error of the unit of weight, corresponding on the average to the mean of 37 observations, is ±0.0067 in terms of the whole amount of light measured, which corresponds to $\pm 0^{m}.0073$ in stellar magnitude. This gives for the probable error of the mean of six observations the value $\pm 0^{m}.018$. For the same quantity Dugan finds, from the residuals from a free-hand curve, the value $\pm 0^{m}.016$. The values of the other constants found by Dugan, which are given above, are also in satisfactory agreement with those here found.

The secondary conjunction comes 13 minutes later than the moment halfway between principal conjunctions. As Dugan has shown (p. 42), this is evidence of a small orbital eccentricity, such that $e \cos \omega = +0.01$, where ω represents, as usual, the longitude of periastron measured from the ascending node of the orbit of the brighter star. The secondary minimum is so shallow that there can be no hope of determining $e \sin \omega$ from the light-curve, and it is legitimate to assume a circular orbit in the calculations which follow.

We have first to rectify the observed light-curve, that is, to remove the effects of ellipticity and the radiation effect, which is done by adding to each observed intensity l the quantity $0.020 (1+\cos\theta)+0.009\, l \cos^2\theta$, and then dividing by 1.023, to reduce the intensity outside eclipse to unity. The rectified intensity at the middle of the secondary minimum is 0.930, which makes the depth of this minimum $0^{m}.079$. The observed and rectified intensities during the principal minimum are given in Table 2. The normals give the mean of 12 observations, except within $1^{h}20^{m}$ of the middle of eclipse, when they depend on 6 observations. The first column gives the mean phase for each of these normals, the second the mean observed intensity, and the third the rectified intensity. It will be observed that the difference between the two amounts to a considerable fraction of the observed intensity near the time of greatest eclipse. The reason for this is that, in recti-

fying the curve, we assume that the fainter side of the companion (which sends us most of the light received at this time) has been increased in brightness until it equals the other side. The fourth and fifth columns of the table give the residuals resulting from the final solutions made on the two hypotheses of uniformly bright star-disks and of disks darkened to zero intensity at the limb.

TABLE 2

Phase	Obs'd Int.	Rectified Int.	Uniform O−C	Darkened O−C	Phase	Obs'd Int.	Rectified Int.	Uniform O−C	Darkened O−C
-2^h53^m3 . . .	0.968	0.989	−0.011	−0.011	$+0^h03^m0$. . .	0.097	0.135	+0.001	+0.002
−2 41.1. . . .	0.967	0.988	−0.012	−0.009	+0 16.0. . .	0.127	0.164	−0.004	−0.004
−2 28.4. . . .	0.942	0.964	−0.027	−0.016	+0 24.4. . .	0.178	0.214	+0.004	+0.003
−2 16.9. . . .	0.920	0.943	−0.008	+0.002	+0 32.2. . .	0.228	0.264	+0.003	+0.003
−2 03.8. . . .	0.878	0.902	+0.002	+0.007	+0 41.7. . .	0.273	0.308	−0.022	−0.020
−1 47.8. . . .	0.787	0.813	+0.004	+0.002	+0 48.1. . .	0.340	0.373	−0.006	−0.004
−1 32.3. . . .	0.688	0.717	+0.007	+0.005	+0 55.1. . .	0.403	0.436	+0.006	+0.004
−1 21.0. . . .	0.608	0.637	+0.010	+0.007	+0 59.9. . .	0.439	0.471	+0.004	+0.003
−1 12.1. . . .	0.527	0.558	−0.004	−0.005	+1 04.4. . .	0.491	0.523	+0.020	+0.018
−1 04.7. . . .	0.466	0.499	−0.008	−0.010	+1 09.0. . .	0.515	0.546	+0.006	+0.006
−0 58.2. . . .	0.424	0.457	+0.003	0.000	+1 13.2. . .	0.538	0.569	−0.001	−0.003
−0 52.2. . . .	0.390	0.423	+0.013	+0.014	+1 17.3. . .	0.567	0.597	+0.005	+0.004
−0 47.0. . . .	0.342	0.376	+0.007	+0.009	+1 26.0. . .	0.650	0.679	+0.016	+0.013
−0 42.5. . . .	0.305	0.339	+0.002	+0.005	+1 36.0. . .	0.707	0.735	+0.001	0.000
−0 37.5. . . .	0.271	0.306	+0.007	+0.008	+1 45.2. . .	0.757	0.784	−0.008	−0.010
−0 32.7. . . .	0.234	0.271	−0.004	−0.005	+1 53.8. . .	0.809	0.835	−0.010	−0.012
−0 28.4. . . .	0.192	0.228	−0.007	−0.007	+2 01.6. . .	0.848	0.873	−0.013	−0.009
−0 22.3. . . .	0.157	0.194	−0.005	−0.005	+2 12.8. . .	0.903	0.927	−0.012	−0.003
−0 15.5. . . .	0.126	0.163	−0.002	−0.002	+2 23.5. . .	0.937	0.959	−0.019	−0.007
−0 08.6. . . .	0.108	0.146	+0.004	+0.003	+2 37.7. . .	0.973	0.994	−0.006	0.000
					+2 57.4. . .	0.976	0.996	−0.004	−0.004

We have now to determine the elements of the system from these data, with the aid of our previous determination of the depth of secondary minimum. The ellipticity constant z, which appears in our equations, may be derived from the quantity c already determined. If the star-disks are uniformly bright, $z=2c=0.018$; if they are completely darkened toward the limb, $z=\frac{5}{4}c=0.011$.

We next transform the observed phases into longitudes in the orbit, by means of the equation

$$\theta=\frac{2\pi t}{P}=0.003215t$$

(in which the phase t is to be expressed in minutes of time), and plot the rectified intensities against $\sin\theta$. The light-curve appears

to be practically symmetrical, the differences between the ascending and descending branches having the aspect of residual errors of observation. Drawing a free-hand symmetrical curve to represent the observations, we find for the rectified intensity at minimum $\lambda=0.133$. Reading from the curve the values of $\sin\theta$ corresponding to a loss of light of $n(1-\lambda)$, we find:

n	0.00	0.25	0.50	0.75
$\sin^2\theta(n)$	0.290	0.1063	0.0528	0.0203

The first of these values, corresponding to the beginning of eclipse, is decidedly uncertain, but the others can be read with accuracy from the curve. From the equation

$$\chi(k, a_0, n)=\frac{\sin^2\theta(n)(1-z\cos^2\theta(\frac{1}{2}))}{\sin^2\theta(\frac{1}{2})(1-z\cos^2\theta(n))}$$

we find, as the observed values of these functions:

$$\chi(k, a_0, \tfrac{3}{4})=0.384\,,\qquad \chi(k, a_0, \tfrac{1}{4})=2.010\,,\qquad \chi(k, a_0, 0)=5.47: \quad (1)$$

We will first determine the elements of the system on the hypothesis that the star-disks appear uniformly bright. To find approximate values, we have the equation:

$$a_0=1-\lambda_1+\frac{1-\lambda_2}{k^2}\,, \qquad (2)$$

in which $1-\lambda_1$ denotes the depth of that minimum during which the larger star eclipses the smaller. To find whether this is the principal or secondary minimum, we must compute a_0 for various values of k, on both assumptions, take $\chi(k, a_0, \frac{1}{4})$ from Table III, and compare with the observed value. Thus we find:

TABLE 3

Large Star in Front at Principal Minimum			Large Star in Front at Secondary Minimum		
k	a_0	$\chi(k, a_0, \frac{1}{4})$	k	a_0	$\chi(k, a_0, \frac{1}{4})$
1.00	0.937	2.296	1.00	0.937	2.296
0.90	0.953	2.127	0.980	0.972	2.324
0.80	0.977	2.002	0.965	1.000	2.362
0.726	1.000	1.900			

To obtain the observed value of $\chi(k, a_0, \frac{1}{4})$ we must have $k=0.809$, $a_0=0.975$. The larger star is in front at the principal eclipse,

which is very nearly total. We have now for the light of the smaller but brighter star:

$$L_2=\frac{1-\lambda_1}{\alpha_0}, \tag{3}$$

whence $L_2=0.890$, and $L_1=0.110$.

We may now compute a light-curve from these constants, using the equations

$$l=1-\alpha L_2\ ; \qquad \sin^2\theta=\frac{A+B\psi(k,\alpha)}{1-z'B\psi(k,\alpha)}\ ; \qquad A+B\psi(k,\alpha_0)=0 \tag{4}$$

in which $z'=\frac{z}{1-z}$, and A and B are constants to be determined. The last equation, which expresses the condition that the obscuration at mid-eclipse shall be α_0, gives $A=1.161B$. Computing l for $\alpha=0.00$, 0.05, etc., and reading the corresponding values of $\sin\theta$ from the free-hand light-curve, we find that a very good representation of them may be obtained by setting $B=0.0310$, whence $A=0.0360$; but on plotting the computed curve on a large scale, and reading off the residuals for the individual observations, it appears that some improvement should be possible.

A differential correction by the method of least-squares was therefore attempted. From equations (3) and (4) we find, neglecting the very small quantity z':

$$l=1-\alpha+\frac{\alpha}{k^2\alpha_0}(1-\lambda_2)\ , \qquad \sin^2\theta=B\{\psi(k,\alpha)-\psi(k,\alpha_0)\}$$

which for brevity we may write

$$\sin^2\theta=B(\psi-\psi_0)\ .$$

Differentiating these, and remembering that λ_2 and $\sin\theta$ (that is, the time of observation) do not vary, we find:

$$\left.\begin{aligned}&(\psi-\psi_0)\frac{dB}{B}+\left(\frac{\delta\psi}{\delta k}-\frac{\delta\psi_0}{\delta k}\right)dk-\frac{\delta\psi}{\delta\alpha_0}d\alpha_0+\frac{\delta\psi}{\delta\alpha}d\alpha=0\\&dl=\left(\frac{1-\lambda_2}{k^2\alpha_0}-1\right)d\alpha-\frac{2\alpha(1-\lambda_2)}{k^3\alpha_0}dk-\frac{\alpha(1-\lambda_2)}{k^2\alpha_0^2}d\alpha_0\end{aligned}\right\} \tag{5}$$

whence, eliminating $d\alpha$, we find dl in terms of $\frac{dB}{B}$, dk, and $d\alpha_0$.

The numerical values of the derivatives of ψ may be found from the tabular differences of the function. Since it appears that the

provisional value of k is a little too great, and that of α_0 too small, these derivatives were computed for $k=0.800$, $\alpha_0=0.978$.

In forming the equations of condition, the observations given in Table 2 were combined into normal places, by taking means of the phases and the residuals for groups of from two to four of these. In all but three cases, the resulting normals depend upon an equal number of observations upon the ascending and descending branches of the curve. Just what observations went to form each normal can be determined by inspection, if it is remembered that the unit of weight for these equations corresponds to the mean of 24 of the original observations—that is, to four of the quantities given in Table II of Dugan's paper between the phases $\pm 1^h 20^m$, and to two of the tabular entries for larger phases. If these equations should be so written that the absolute terms were expressed in light-intensity, they would be of very different weights, for the observations, whose probable error in stellar magnitude varies but little, give the intensity with much greater precision when the star is faint. To allow for this, each equation is divided through by the corresponding rectified intensity, so that the absolute terms are of the form $\frac{dl}{l}$, and may be converted into residuals in stellar magnitude by changing their signs, and multiplying them by 1.08. The equations so obtained are given weights proportional to the number of observations. The use of the rectified, rather than the observed intensity, in this reduction, is equivalent to giving the observed magnitudes a weight which diminishes as the star becomes fainter, being 1.00 at maximum (mag. 10.5), 0.85 at $11^m\!.5$, 0.66 at $12^m\!.5$, and 0.51 at minimum ($13^m\!.0$). This appears to be justified by Dugan's remark, "When there was haze, or dew, or moonlight, observations at faintest light were mere guesswork" (*op. cit.*, p. 41), and by the fact that under these conditions he usually stopped observing.

Table 4 gives these equations, with the residuals remaining after their solution, and also the residuals resulting from the solution to be described later, in which the star-disks are supposed to be completely darkened toward the limb. To make the equations

more nearly homogeneous, the unknowns have been taken as $x=\frac{1}{2}\frac{dB}{B}$, $y=dk$, $z=3da_0$.

TABLE 4

EQUATIONS OF CONDITION

Weight	Mean Phase		Uniform O−C	Darkened O−C
½	$0^h\ 5^m.8$	$-0.15x-1.72y-2.49z=+0.008$	+0.017	+0.017
½	0 15.8	$-0.42x-0.93y-1.95z=-0.030$	−0.017	−0.018
1	0 26.9	$-0.76x-0.23y-1.27z=-0.020$	−0.005	−0.006
1	0 38.6	$-0.92x-0.05y-0.75z=-0.023$	−0.013	−0.008
1	0 50.6	$-0.98x-0.11y-0.48z=+0.007$	+0.012	+0.014
1	1 01.8	$-1.00x-0.20y-0.32z=+0.009$	+0.010	+0.005
1	1 12.9	$-0.99x-0.29y-0.24z=+0.005$	+0.003	0.000
1	1 23.5	$-0.96x-0.36y-0.18z=+0.024$	+0.020	+0.014
1	1 34.2	$-0.92x-0.41y-0.14z=+0.011$	+0.006	+0.003
1½	1 48.9	$-0.84x-0.45y-0.09z=+0.001$	−0.006	−0.009
1	2 02.7	$-0.73x-0.43y-0.06z=+0.001$	−0.006	−0.001
1	2 14.8	$-0.62x-0.40y-0.04z=-0.004$	−0.010	−0.001
1	2 26.0	$-0.44x-0.30y-0.02z=-0.020$	−0.025	−0.012
1	2 39.4	$0.00x\ \ 0.00y\ \ 0.00z\ \ -0.009$	(−0.009)	−0.005

The normal equations resulting from these, and their solution, are:

$$8.39x+3.12y+3.76z=-0.0015 \qquad x=-0.0008 \quad \text{weight } 5.4$$
$$3.12x+3.13y+3.79z=+0.0029 \qquad y=-0.0163 \quad \text{weight } 1.0$$
$$3.76x+3.79y+7.63z=+0.0489 \qquad z=+0.0149 \quad \text{weight } 3.0$$

The weighted sum of the squares of the residuals is diminished by the solution from 0.002681 to 0.001987. The resulting value of the probable error of the unit of weight is ± 0.0095, corresponding, in stellar magnitude, to ± 0.0104.

Applying the corrections thus derived to the provisional constants, we have the corrected values:

$$B=0.03095\pm 0.00026, \text{ whence } 1-\lambda_1=0.868$$
$$k=0.793 \quad \pm 0.010 \qquad L_2=0.886$$
$$a_0=0.980 \quad \pm 0.002 \qquad L_1=0.114$$

The light-curve computed from these data gives values for the residuals which agree, within the limits of error of the graphical process, with those derived from the equations of condition. Hence the values of the constants just found may be accepted as final.

For the beginning of eclipse we find $\sin^2\theta' = 0.2269$, corresponding to a semi-duration of $2^h 34^m.6$. To find the remaining elements, we have the equations:

$$\left.\begin{array}{l} a_1^2(1-z\cos^2\theta')(1+k)^2 = \cos^2 i\cos^2\theta' + \sin^2\theta'; \quad b_1^2 = a_1^2(1-z) \\ a_1^2(1-z)\{1+kp(k,\alpha_0)\}^2 = \cos^2 i\ ; \quad a_2 = ka_1\ ; \quad b_2 = kb_1 \end{array}\right\} \quad (6)$$

From Table I we find $p(k, \alpha_0) = -0.934$, and then

$$a_1 = 0.2695\ , \qquad a_2 = 0.2137\ , \qquad \cos i = 0.0695\ .$$

To find the probable errors of these quantities, we must express their differential increments in terms of the quantities x, y, z which appear in our least-squares solution, and also make allowance for the fact that the probable errors of these three quantities are not determined independently of one another. From the form of the normal equations, we have for the sum of the squares of the residuals (if x, y, z denote variations from the values which make the sum of the squares a minimum M):

$$[pvv] = M + 8.39x^2 + 6.24xy + 7.02xz + 3.13y^2 + 7.08yz + 7.63z^2\ ,$$

or

$$[pvv] = M + (2.90x + 1.08y + 1.30z)^2 + (0.98y + 2.42z)^2 + (1.005y)^2\ .$$

The quantities in parentheses are determined independently by the observations, with weight unity. We will call them p, q, r. If the small quantity z is neglected, a_1, a_2, and i are given by the equations:

$$a_1^2(1+\cot^2 i) = \frac{B}{\phi_1(k)}\ ; \quad \cot^2 i = \frac{B}{\phi_2(k)} - A\ .$$

$$a_2 = ka_1 \qquad\qquad A + B\psi(k, \alpha_0) = 0\ .$$

These are abundantly accurate enough for our present purpose (giving, in fact,

$$a_1 = 0.268\ , \qquad a_2 = 0.212\ , \qquad \cos i = 0.071)\ .$$

Differentiating them, and introducing the numerical values

$$\phi_1(k) = 0.431\ , \quad \phi_2(k) = 0.749\ , \quad \frac{\delta\phi_1}{\delta k} = -0.01\ , \quad \frac{\delta\phi_2}{\delta k} = +0.93$$

(which are readily found from Table II*a*), and also

$$\psi(k, \alpha_0) = -1.17\ , \quad \frac{\delta\psi}{\delta k} = +0.44\ , \quad \frac{\delta\psi}{\delta\alpha_0} = -2.18\ ,$$

we find without difficulty:

$$\frac{da_1}{a_1}=1.99x+0.06y+0.02z=0.69p-0.36q-0.33r\,.$$

$$\frac{da_2}{a_2}=1.99x+1.32y+0.02z=0.69p-0.36q+0.93r\,.$$

$$d(\cot^2 i)=0.010x-0.040y-0.024z=0.004p-0.012q-0.032r\,.$$

The weights of these three quantities are the reciprocals of the sum of the squares of the coefficients of p, q, r, in these equations.

The final values of the elements thus derived on the hypothesis of uniformly bright star-disks are given below, in Table 6, and the outstanding residuals for the observations and normal places above, in Tables 2 and 4. These residuals are distinctly systematic, and are larger than might have been expected, giving a probable error of $\pm 0^{m}.0104$ for the mean of 24 of the original observations, or of $\pm 0^{m}.021$ for the mean of 6, while from the observations outside principal minimum we found $\pm 0^{m}.018$ for this latter quantity. The discrepancies between observation and calculation, and especially the large negative residuals near the beginning and end of eclipse—which indicate that the actual duration is longer than the computed—are such as would be caused by darkening of the star-disks toward the limb.

We therefore proceed to a second solution, in which we assume that the apparent brightness of the disks varies as the cosine of the angle of emission of the light, and falls off to zero at the limb. We must now use the formulae and tables prepared for this case of completely darkened stars.

For the relation between k and α_0 we now have:

$$Q(k,\alpha_0)=\frac{1-\lambda_2}{\alpha_0-(1-\lambda_1)}\,.$$

If we suppose the larger star to be in front at principal minimum (so that $1-\lambda_1$ is greater than $1-\lambda_2$), we compute Q for various values of α_0, find the corresponding values of k from Table V,[1] and then take the χ-functions from Table IIIx, with arguments k, α_0. If, however, we assume that the smaller star is in front, we must

[1] It aids in this process to note that $Q(k, \alpha_0)-k^2$ changes but slowly with k and α_0.

interchange the numerical values of λ_1 and λ_2, compute Q and k as before, and α_0'' by the equation

$$\alpha_0'' = \alpha_0 \frac{Q(k, \alpha_0)}{Q(k, 1)}$$

and take the χ-functions from Table III*y*, with arguments k, α_0''.

In this way we find (assuming $1-\lambda_1=0.867$, $1-\lambda_2=0.070$):

TABLE 5

LARGE STAR IN FRONT

α_0	$Q(k, \alpha_0)$	k	α_0''	$\chi(k, \alpha_0, \frac{3}{4})$	$\chi(k, \alpha_0, \frac{1}{2})$	$\chi(k, \alpha_0, 0)$
1.000	0.569	0.689	....	0.502	1.686	3.53
0.980	0.619	0.740	....	0.459	1.773	3.93
0.950	0.844	0.884	....	0.412	1.905	4.62
0.940	0.959	0.961	....	0.389	1.965	4.89
0.937	1.000	1.000	....	0.373	2.012	5.09
			SMALL STAR IN FRONT			
0.950	0.985	0.982	0.942	0.371	2.024	5.17
0.980	0.953	0.950	0.968	0.365	2.048	5.30
1.000*	0.934	0.925	1.000	0.360	2.080	5.49
$1+x$†	0.934	0.915	$1+x$	0.357	2.090	5.51
Observed values of χ-functions....			....	0.384	2.010	5.47:

* Grazing annular eclipse. † Central annular eclipse.

It appears from this table that the smaller star must undergo eclipse at the principal minimum. The observed value of $\chi(k, \alpha_0, \frac{3}{4})$ is exactly represented if $\alpha_0=0.939$, $k=0.972$, and that of $\chi(k, \alpha_0, \frac{1}{2})$ if $\alpha_0=0.937$, $k=0.998$. That of $\chi(k, \alpha_0, 0)$ is too uncertain to be of value in this connection.

We therefore assume $k=0.985$, $\alpha_0=0.938$—whence $L_2=0.925$, $L_1=0.075$—and compute a light-curve from these elements, using formula (4), but taking the ψ-functions from Table II*x*, and using the value $z=0.011$, already found to be appropriate in this case. We thus find $B=0.0306$, $A=0.0330$, and obtain a curve which is already very good but appears capable of some improvement. In this case the adjustment was made by empirical changes of the constants, as it did not seem worth while to undertake the labor of another least-squares solution. A very satisfactory representation

was obtained by diminishing k by 0.005, B by 0.0002, and λ_1 by 0.002, leaving α_0 unchanged. These changes reduce the depth of the secondary minimum by $0^{m}.002$, which is practically negligible.

The residuals outstanding from the light-curve finally adopted are given in the last columns of Tables 2 and 4. In the latter, the weighted sum of the squares of the residuals is 0.001126, as against 0.001528 for the preliminary darkened solution. Both of these sums include the normal place at phase $2^{h}39^{m}.4$, which falls outside the eclipse on the uniform hypothesis. If this observation is included in the uniform case also, the sum of the squares is 0.002068 for the best possible uniform solution—almost twice as great as on the assumption of darkening at the limb. That this star does not present a uniform disk seems, therefore, to be beyond question.

The residuals from the darkened curve still show a systematic run of signs, and are similar in character to, though less in numerical magnitude than, those of the uniform solution. This would indicate that the apparent brightness of the star's disk actually falls off still more rapidly than we have assumed. It would, however, hardly be safe to draw this conclusion for the representation of the observations is already all that could be expected—the probable error of the mean of 24 observations, according to the darkened solution, being $\pm 0^{m}.0073$, and that of 6 observations $\pm 0^{m}.015$, while the latter quantity for the observations outside principal minimum was $\pm 0^{m}.018$.

Proceeding to find the elements corresponding to the darkened curve, we have again a set of equations of the form (6), in which now the function $p(k, \alpha_0)$ must be taken from Table Ix. We find, from the computation of the light-curve, $\sin^2 \theta' = 0.2642$—corresponding to a semi-duration of eclipse of $2^{h}48^{m}.0$—and then $a_1 = 0.262$, $a_2 = 0.257$, $\cos i = 0.054$.

The final elements derived for the system of *Z Draconis*, on the two hypotheses of uniform disks and disks completely darkened toward the limb, are summarized in Table 6, along with various related quantities. The probable errors of the darkened elements are not given, since these were not obtained by a formal least-squares solution; but they are presumably somewhat less than those of the uniform elements. The elements found by Dugan

(*op. cit.*, p. 43) are also given, for comparison. These differ so little from the final elements derived on the hypothesis of uniform star-disks that the only return for the labor of the least-squares solution has been the certainty that the darkened curve fits the observations better than the other can possibly do.

TABLE 6

Elements of the System of *Z Draconis*

	Uniform			Darkened
	Dugan	Least-Squares		
Maximum radius of larger star....... a_1	0.270	0.2695	±0.0020	0.262
Minimum " " " " .... b_1	0.266	0.2671		0.261
Maximum " " smaller " .... a_2	0.217	0.2137	±0.0025	0.257
Minimum " " " " .... b_2	0.214	0.2118		0.256
Ratio of the radii of the stars......... k	0.805	0.793	±0.010	0.980
Ratio of the axes of the spheroidal stars......................... $1+\frac{1}{2}z$	1.016	1.009	±0.006	1.0055
Least apparent distance of centers...cos i	0.074	0.0695	±0.0024	0.054
Inclination of orbit plane............. i	85°44′	86°01′	±7′	86°55′
Eccentricity of orbit............ $e \cos \omega$	+0.010	+0.010	±0.005	+0.010
Maximum fraction of light of the smaller star obscured during eclipse........ α_0	....	0.980	±0.002	0.938
Difference of light of the sides of the larger star...................... $2b$	0.032	0.040	±0.006	0.040
Light of the smaller star............. L_2	....	0.886		0.927
Light of the larger star				
Brighter side.................... L_1	....	0.114		0.073
Fainter side................. L_1-2b	....	0.074		0.033
Ratio of surface-brightness				
Of the bright sides of the two stars . $\frac{J_2}{J_1}$	15.8	12.3		13.1
Of the sides of the fainter star........	1.58	1.54		2.2
Stellar magnitude				
Of the brighter star.................	....	10.57		10.50
Of the fainter star				
Bright side......................	....	12.80		13.26
Faint side.......................	....	13.27		14.12
Density of the brighter star.......... ρ_2	0.36	0.39		0.22
Density of the fainter star........... ρ_1 (assuming equal masses)	0.19	0.19		0.21

The unit of light is the combined light of the brighter sides of the two stars, and corresponds to a stellar magnitude of 10.24. The unit of length is the mean radius of the orbit, which must

remain unknown, as the star is far too faint for spectrographic study. The unit of density is the sun's density. It is probable, by analogy with investigable cases, that the brighter component is more massive than the other, and therefore that it is denser, and the fainter component less dense, than the tabular values.

The general characteristics of *Z Draconis* are fairly typical of eclipsing variables. There is nothing unusual about the system, except the great accuracy of the observed light-curve, and the consequent precision of the elements. There can be little doubt that the brighter component, at least, is considerably darkened at the limb, and the eclipse theory, on this hypothesis, gives a very satisfactory account of the observed variations in brightness. The principal difference between the uniform and darkened elements is that the smaller and brighter component comes out considerably larger and less dense on the latter assumption (as is almost always the case). Even so, it is decidedly denser than the majority of eclipsing variables.

In response to an inquiry from the writer, Miss Cannon very kindly examined the spectrum of *Z Draconis* on several plates, but found it too faint to classify. From analogy with other eclipsing variables of similar density, it might be expected to be of class A or class F.

While the data are not sufficient to justify an estimate of the distance of this system, they are enough to make it probable that it is very remote. Assuming the sun's stellar magnitude to be -26.8, it is easy to show that a star of magnitude m, mass M, density ρ, and surface brightness J (with reference to the sun as a standard) must have the parallax $\pi'' = (0.630)^{m+0.2}\rho^{\frac{1}{3}}M^{-\frac{1}{3}}J^{-\frac{1}{2}}$.[1] Applying this to the bright component of *Z Draconis*—using the data of the darkened solution—we find,

$$\pi'' = 0\overset{''}{.}0044 M^{-\frac{1}{3}}J^{-\frac{1}{2}} .$$

The mass and surface brightness are unknown; but in most eclipsing variables, when they can be estimated, they are found to be greater than in the case of the sun. It seems likely, therefore, that the distance of this system is of the order of magnitude of 1,000 light-years.

[1] Cf. Schlesinger, *Publications of the Allegheny Observatory*, **2**, 61, 1910.

II. The series of photometric measures of *RT Persei* by Dugan, published in No. 1 of the *Contributions from the Princeton University Observatory*, comprises nearly fifteen thousand comparisons, and fixes the light-curve with a precision that is not surpassed by observations on any other star. The unusual accuracy of the comparisons and the existence of a well defined secondary minimum have permitted the most complete application of the theory of eclipsing binaries. For this star more information has been derived from the light-curve than has ever before been obtained from a single curve—in fact all the quantities have been derived that are possible from photometric observations alone. In addition to the nine independent unknown quantities found for *Z Draconis*, the line-of-sight component of the eccentricity and a definite periastron effect are determined from the curve of *RT Persei*, and in this case also the existence of darkening at the limb is demonstrated. The method of discussion closely follows that for *Z Draconis*, and only those points which are more or less unique will be mentioned in detail in this presentation. It may, in fact, facilitate matters if the points of special interest in the discussion are simply enumerated in the order in which they arose.

1. The asymmetry of the light-curve at principal minimum becomes practically negligible after a shift of the epoch of mid-eclipse of -1.0 minute. The sum of the residuals from the computed curves is reduced 20 per cent by such an adjustment. With this slight change the formula for minima derived by Dugan is $2417861^{d}.63026+0^{d}.8494222\,E$. The matter of the asymmetry outside the minima will be referred to later.

2. The normal places of six observations each, published in Table IV (*op. cit.*), were combined into supernormals of 30 observations each for maximum light, of 24 observations each for secondary minimum, and of 12 observations each for the primary. From the maximum-light groups 13 equations of condition were formed and values were derived for ellipticity, differential reflection, and mean maximum light. All observed intensities were then rectified by means of the relation:

$$\text{Rectified intensity}=\frac{\text{Observed intensity}+b(1+\cos\theta)+cl\cos^2\theta}{L_a},$$

where from the least-squares solution $b=0.011\pm0.0046$ and $c=0.017\pm0.012$; the divisor, $L_0=1.015$, reduces the rectified intensity outside minima to unity. The maximum-light groups are given in Table 7. The phases are computed from the nearest primary minimum, and the first column of residuals refers to the mean maximum light of 1.000.

TABLE 7

Adjusted Phase	Observed Intensity	Rectified Intensity	$O-C_1$	$O-C_2$
			L	L
$+2^h 11^m$	0.983	0.998	−0.002	−0.008
2 39	0.995	1.006	+0.006	−0.001
3 24	0.993	0.998	−0.002	−0.010
4 27	1.017	1.017	+0.017	+0.007
5 32	1.015	1.010	+0.010	0.000
6 58	1.017	1.012	+0.012	+0.004
+7 54	1.012	1.010	+0.010	+0.004
−8 1	0.995	0.992	−0.008	−0.002
6 47	0.989	0.983	−0.017	−0.009
5 24	0.995	0.990	−0.010	0.000
4 17	1.000	1.000	0.000	+0.010
3 22	0.978	0.984	−0.016	−0.008
−2 28	0.991	1.004	+0.004	+0.010

3. As noted by Dugan, the intensity is distinctly higher between primary and the following secondary than in the half of the curve preceding the primary. Maximum intensity in the latter case, determined from 180 observations, is 0.992 ± 0.002. During the first half of the maximum light 210 observations give the mean value 1.007 ± 0.002. The sum of the squares of the residuals in the foregoing table is reduced by applying the appropriate sine curve correction[1] from 1402 to 595. The amplitude of the sine curve is 0.020. From the residuals in the last column above the probable error of the mean maximum light is ±0.0015 after this adjustment; it was ±0.004 from the least-squares solution.

Since the middle point of secondary eclipse comes nine minutes earlier than the point midway between primary minima, the periastron passage occurs between primary eclipse and the following secondary, and therefore the asymmetry just discussed is evidently

[1] *Astrophysical Journal*, **36**, 69, 1912.

to be attributed to the periastron effect. The difference of two-hundredths of a magnitude should probably be considered the result of secondary heating and reflection arising from the eccentricity of the orbit, or the result of a secondary tidal action. It has been shown by Dugan (*op. cit.*, p. 40) that simple reflection alone is not sufficient to account for the mean reflection effect considered in the last section. In like manner, considering the small eccentricity found later, it is immediately evident that the increase in the differential reflection at periastron is insufficient to account for the observed difference, and recourse must be had again to the reasonable supposition that there exists a cumulative heating effect.

4. The solution for the orbit was made completely by the graphical method, experience having shown that the adjustment by least squares is an unnecessary refinement. The circular elements for uniform disks obtained by Dugan served as a first approximation to the interpretation of the principal eclipse. The main difference between the new circular uniform elements and those already published arises from the use of the new ellipticity constant. The two solutions are compared below:

	Uniform Circular Elements Derived from Principal Minimum						
	k	a	b	i	L_b	L_f	$\frac{J_b}{J_f}$
Dugan	1.00	0.274	0.268	85°38′	0.861	0.139	6.2
Shapley	1.00	0.272	0.267	85°44′	0.839	0.161	5.2

These sets of elements represent the primary accurately, but they are not adequate for the secondary eclipse which is noticeably of longer duration.

5. From the displacement of secondary minimum we derive:

$$e \cos \omega = \frac{\pi\left(t_2 - t_1 - \frac{P}{2}\right)}{P(1+\operatorname{cosec}^2 i)} = -0.012 .$$

The line-of-sight component of the eccentricity is obtained from the equation:

$$e \sin \omega = \frac{(p_s - p_p)k}{2+(p_s+p_p)k} ,$$

where the quantities p_s and p_p are the functions $p(k, \alpha_0)$ taken from Table I for $k=1.00$ and for the values of the maximum eclipse, α_0, at secondary and primary minima, respectively. The value of the fraction of the light eclipsed at the primary, α_{0p}, used in the circular solution above, is 0.823. The fraction at the secondary eclipse would be the same for a circular orbit; but it is in general different when the orbit is not circular, and in this particular case must be smaller since the relative widths of the two minima indicate that the stars are nearer apastron at secondary eclipse.

For an eccentric orbit the depths of the two minima and the maximum percentages of light eclipsed are connected by the relation (when $k=1$):

$$\alpha_{0s}=\frac{\alpha_{0p}(1-\lambda_s)}{\alpha_{0p}-(1-\lambda_p)}.$$

A few approximations showed that for $1-\lambda_s=0.13$, $1-\lambda_p=0.69$, and consequently $\alpha_{0s}=0.807$, the light-curve computed for the secondary minimum was entirely satisfactory. Its duration is exactly four hours, and is 20.6 minutes longer than the primary. Adopting these values of α_{0p} and α_{0s} we derive immediately $e \sin \omega =+0.043$, $e=0.045$, and $\omega=164°$.

6. The light-curve for the secondary minimum was computed by transforming the circular elements of principal minimum into another set of circular elements for secondary minimum by means of the equations (30) and (32), *Astrophysical Journal*, **36**, 57, and working the problem in reverse order from elements to light-curve. By means of equation (30) the final elliptic elements were readily derived and are given in a subsequent table. The light of the two stars and the relative surface intensities are obtained from the relations:

$$L_b=\frac{1-\lambda_p}{\alpha_{0p}}=1-L_f=\frac{L_fJ_b}{J_f}=0.839\,.$$

7. The solution for darkened elements of *RT Persei* developed a novelty in the apparent existence of a non-central annular eclipse at principal minimum. This condition is to be expected upon an examination of the uniform elements. Experience has shown that

the general effect of the introduction of the assumption of complete darkening at the limb is to increase the inclination of the orbit and

TABLE 8

Adjusted Phase	Observed Intensity	Rectified Intensity	Uniform O−C	Darkened O−C
			L	*L*
$-2^h\ 2^m.6$	0.980	0.996	−0.004	−0.004
1 47.7	0.963	0.980	−0.015	−0.010
1 35.5	0.928	0.949	−0.013	−0.012
1 23.7	0.891	0.911	+0.007	+0.003
1 11.5	0.795	0.816	−0.013	−0.017
1 0.5	0.727	0.749	0.000	−0.004
0 51.9	0.664	0.686	+0.006	0.000
0 44.2	0.593	0.615	0.000	−0.002
0 33.9	0.508	0.530	+0.003	+0.008
0 24.5	0.421	0.444	−0.008	+0.003
0 18.4	0.377	0.399	−0.003	+0.009
0 10.3	0.330	0.353	+0.007	+0.013
−0 2.5	0.291	0.314	0.000	−0.003
+0 4.2	0.294	0.316	−0.002	−0.005
0 12.1	0.328	0.351	−0.005	+0.003
0 19.7	0.378	0.400	−0.013	+0.001
0 26.9	0.432	0.454	−0.018	−0.007
0 34.4	0.505	0.527	−0.003	+0.001
0 41.6	0.566	0.588	−0.004	−0.005
0 48.5	0.626	0.648	0.000	−0.007
0 55.6	0.694	0.716	+0.005	0.000
1 3.1	0.758	0.780	+0.010	+0.005
1 12.0	0.824	0.846	+0.013	+0.007
1 22.2	0.871	0.891	−0.004	−0.009
1 32.0	0.950	0.970	+0.022	+0.022
1 42.2	0.960	0.979	−0.005	−0.003
1 51.3	0.968	0.985	−0.015	−0.010
+1 59.3	0.995	1.011	+0.011	+0.011
SECONDARY MINIMUM				
−2 12.1	1.017	1.013	+0.013	+0.013
1 41.8	0.985	0.983	−0.007	−0.005
1 7.5	0.946	0.946	−0.008	−0.009
0 44.8	0.923	0.924	+0.001	+0.002
−0 17.7	0.888	0.890	+0.008	+0.007
+0 10.9	0.877	0.879	+0.003	0.000
0 34.3	0.895	0.896	−0.010	−0.009
0 57.6	0.938	0.939	−0.003	−0.002
1 26.4	0.970	0.970	−0.007	−0.007
+1 56.2	0.986	0.985	−0.014	−0.010

invariably to increase the size of the brighter star, usually without appreciable change in the size of the darker companion.[1] In the

[1] *Astrophysical Journal*, **36**, 281, 1912.

uniform solution the stars are found to be sensibly equal, with large partial eclipses at both minima. An increase of 20 per cent in the radius of the brighter star (as was found for *Z Draconis*) with a small increase in the inclination of the orbit would project the darker companion entirely upon the disk of the larger star at the middle phase, of primary eclipse. This general result is so definite that we can safely assume at once that the secondary eclipse is total and the loss of light at that phase is $1-\lambda_s=1-L_b=L_f=0.121$, the light of the smaller star. Also $\alpha_{os}=1$, but since the orbit is eccentric (we adopt here the values of e and ω derived from the uniform solution) the notation of equation (3), *Astrophysical Journal*, **36**, 390, should be altered for this occasion to allow for the difference in the percentages of eclipse at the two minima, i.e.:

$$Q(k, \alpha_{op})=\frac{1-\lambda_p}{\alpha_{os}-(1-\lambda_s)}\,.$$

The rectification of the intensity-curve for uniform disks suffices for the darkened case, but $z_d=\frac{5}{8}z_u=\frac{5}{4}c=0.021$. The darkened curve for the same depths near minima is so much less pointed than the uniform that it is now better to take the ranges of variation smaller; but it is to be observed that the annular eclipse has no constant minimum phase and resembles a partial eclipse very closely. For $1-\lambda_p=0.682$ and $1-\lambda_s=0.121$, $Q(k,\alpha_{op})=0.777$. It is obvious that α_{op} must lie between unity, which corresponds to a grazing annular eclipse, and $1+x$, corresponding to central transit. A value nearer the latter limit is to be expected, since the inclination approximates 90° and the secondary eclipse near apastron is total. Assuming the values $1+0.4x$, $1+0.6x$, $1+0.8x$, $1+x$, the corresponding k was taken from Table V and the light-curve for the principal minimum computed. The best representation was found for $\alpha_{op}=1+0.8x$, $k=0.80$. The light-curve for the secondary minimum and the elliptical elements were computed in the same manner as for the uniform orbit. For $k=0.80$, Table Iy gives $\alpha_{op}=1.038$, which is the greatest loss of light at primary eclipse in terms of the loss of light at internal tangency.

8. The evidence of the deviations from the computed curves is decidedly in favor of darkening toward the limb. The sums of the squares of the residuals are as follows:

	Uniform	Darkened
Principal minimum	2507	1938
Secondary minimum	710	562

The small, but apparently systematic deviations at principal minimum of the observed curve from the computed uniform curve occur at the same phases and are in the same direction as the deviations of the computed darkened curve from the uniform. Moreover, a mere inspection shows that no adjustment of the uniform curve can possibly conform so closely to the observations as the darkened solution.

TABLE 9

ELEMENTS OF THE SYSTEM OF *RT Persei*

	Uniformed	Darkened
Maximum radius of brighter star a_b	0.283	0.333
Minimum " " " " b_b	0.278	0.329
Maximum " " fainter " a_f	0.283	0.266
Minimum " " " " b_f	0.278	0.263
Ratio of radii of the stars k	1.000	0.800
Ratio of the axes of the spheroidal stars $1+\frac{1}{2}z$	1.017	1.010
Least apparent distance of centers $\cos i$	0.0805	0.0286
Inclination of orbit plane i	85°23′	88°22′
Eccentricity of orbit e	0.045	0.045
Longitude of periastron ω	164°	164°
Maximum loss of light at primary minimum* α_{op}	0.823	1.038
Maximum loss of light at secondary minimum* α_{os}	0.807	1
Difference of light of the sides of the fainter star $2b$	0.022	0.022
Light of the brighter star L_b	0.839	0.879
Light of the fainter star		
Brighter side L_f	0.161	0.121
Fainter side L_f-2b	0.139	0.099
Ratio of surface brightness		
Of the bright sides of the two stars $\frac{J_b}{J_f}$	5.2	4.6
Of the sides of the fainter star	1.16	1.22
Mean maximum intensity		
Near periastron	1.007	1.007
Near apastron	0.992	0.992
Density of the brighter star ρ_b	0.43	0.26
Density of the fainter star ρ_f (assuming equal masses)	0.43	0.50

* In terms of the loss of light which would occur at the moment of internal tangency.

The observational groups are given in Table 8 (p. 424), the phases being referred to the middle points of the minima.

9. In Table 9 the final uniform and darkened elliptic elements and various related quantities are given. The units are the same as those employed for *Z Draconis*. The stellar magnitude of *RT Persei* is 10.63 at maximum; the spectral type is F? The computed parallax is $0.0041 M^{-\frac{1}{3}} J^{-\frac{1}{6}}$, indicating that the distance is of the same order of magnitude as that of *Z Draconis*.

PRINCETON UNIVERSITY OBSERVATORY
December 15, 1913

THE ILLUMINATION-CURRENT RELATIONSHIP IN POTASSIUM PHOTO-ELECTRIC CELLS

By HERBERT E. IVES

SYNOPSIS

1. HISTORICAL

The photo-electric current is a function of voltage, of electrode distance, of the kind of gas between electrodes, of pressure, and of illumination.

The *voltage-current* relation and the *electrode-distance-current* relation, under constant illumination, were the subject of study by Stoletow[1] and others some twenty years ago.

The effects of pressure variations and of different gases, again under constant illumination, were studied by Stoletow, Varley, and other investigators. All of these studies revealed a complicated relationship between the current and the variables in question, for which qualitative explanations have been offered in terms of saturation, ionization by collision, etc.

[1] *Journal de physique*, ii, **9**, 468, 1890.

One of the most important relationships to be determined, both from its theoretical bearing and from its practical application, is that between photo-electric current and intensity of illumination.

Elster and Geitel[1] concluded from some early experiments that the current is directly proportional to the illumination. Lenard[2] reached the same conclusion. In his results, however, the fact on which greatest emphasis is laid is that the final voltage acquired by the sensitive surface is a constant, independent of the intensity of illumination, while the current or rate of acquisition of voltage is certainly not a constant. Actually his figures show a current increasing more rapidly than the illumination. Lenard, accepting the linear illumination-current relationship as proved, has recently used alkali metal cells for measuring the decay of light in phosphorescence.

Griffith,[3] working with a zinc plate, illuminated by a spark, made careful correction for air absorption and measured the current by a balancing method in which the electrometer served merely as a detector. His results plotted take the form of a curve concave toward the current axis (similar to Figs. 8–10 and Lenard's values).

Richtmyer,[4] using a sodium cell, found a strictly linear relation between illumination and current, over an enormous range. He suggested various laboratory applications of the cell for photometric work.

Elster and Geitel,[5] in a paper appearing after a large part of the work here described was completed, found the photo-electric current, in cells having a gas atmosphere of a fraction of a millimeter, directly proportional to the illumination over an even greater range than Richtmyer investigated. They have developed a special form of cell, having a surface of colloidal metal or hydride, with an atmosphere of inert gas at a pressure determined by them as giving great sensibility. These cells are now obtainable on the market.

[1] *Annalen der Physik*, **48**, 625, 1893.
[2] *Ibid.*, **8**, 149, 1902.
[3] *Philosophical Magazine*, **14**, 297, 1907.
[4] *Physical Review*, **29**, 71, 404, 1909.
[5] *Physikalische Zeitschrift*, **14**, 741, 1913.

2. OBJECT OF PRESENT STUDY

It appeared proved from the work of Elster and Geitel and of Richtmyer (which appeared subsequently to that of Griffith) that the photo-electric current is truly proportional to intensity of illumination. The extreme sensibility of the alkali metals is well established. Elster and Geitel, and later Pohl and Pringsheim[1] have shown the sensitiveness of the metals sodium, potassium, rubidium, and caesium to extend well down into and through the visible spectrum, the maximum (of the "selective" effect) lying at progressively greater wave-lengths in the order in which the metals are above given.

It appeared to the writer to be desirable at this time to study thoroughly the alkali metal cell as a possible substitute for the eye in photometry, particularly in colored-light photometry. If it should be possible to produce cells of uniform wave-length sensibility, to develop a colored absorbing screen which should make the resultant spectral-sensibility curve that of an average eye,[2] then it should be possible to tie down to a purely physical instrument the characteristics of that wonderful, but most troublesome, physiological one—the human eye. The work was therefore undertaken as the logical continuance of the writer's study of heterochromatic photometry.

In order for a physical photometer to be available for anything except as a detector in a null method with lights of the same color, or for the measurement of lights of the same color where the intensity-response relationship has been determined, the relationship between the intensity of illumination and the resultant current or reaction must be a simple one, the same for all colors. The most desirable relationship, as well as the simplest, is the linear one, which has been credited to the photo-electric cell. Granted that this simple relationship exists, the investigation as planned was to have been chiefly directed to the questions of the method of construction, performance, and reproducibility of the cells, their behavior under various photometric tests, the choice of alkali metal, and the specification of the proper color screen.

[1] *Berichte der Deutscher Phys. Ges.*, **12**, 215, 349, 1910, and subsequent papers.

[2] Ives, "Photometry of Lights of Different Colors," *Phil. Mag.*, **24**, 149, 352, 744, 846, 854, 1912.

As will appear, the work took a different direction. The mode of construction of the cells and the methods of using them as heretofore described were not found satisfactory. Finally when the first difficulties were overcome the illumination-current relation was not found to be linear in photo-electric cells as heretofore constructed.

3. PRELIMINARY APPARATUS AND RESULTS

Five methods of measuring photo-electric current are to be found in the literature: (1) by the rate of drift of an electrometer needle; (2) by the ballistic method, or the charge acquired in a definite exposure time by an electrometer connected to the cell; (3) by measuring the potential across the terminals of a high resistance in series with the cell; (4) by balancing the photo-electric current with a current variable in a known manner, using either an electrometer or sensitive galvanometer as a detector; (5) by the deflection of a sensitive galvanometer.

As a large part of the problem was anticipated to be the study of the wave-length sensibility-curves, the comparatively insensitive galvanometer method was not considered. A Dolazalek electrometer (made by the Cambridge Scientific Instrument Co.) was accordingly employed throughout the work. In order in starting to have the benefit of all previous work, a cell was purchased on the market. The metal was potassium, the cell had a quartz window (which was not necessary in this work) and was made by Müller-Uri. It is shown in the diagram, Fig. 6, *a*.

The arrangement of cell, electrometer, and light-source was copied closely from that described by Richtmyer[1] and is shown in Fig. 1. The cell and electrometer were placed on a metal shelf on a brick pier in the laboratory basement; a galvanized-iron cover fitted over them, pierced with openings for a set of keys[2] for connecting and disconnecting cell, electrometer, and known e.m.f. for calibration purposes. Glass windows permitted the exciting light and that illuminating the electrometer mirror to enter. In a separate iron box, connected to the first by metal tubes, was a six-volt

[1] *Loc. cit.*

[2] See McClung, *Conduction of Electricity through Gases and Radioactivity.*

storage cell, discharging through a 2500-ohm resistance, from which various voltages might be taken by a sliding contact. This latter was connected to the potassium and served to neutralize the contact e.m.f. which in the dark causes a strong current. More will be said about this later.

The needle was charged to a potential of 100 volts by contact with a set of dry batteries. The needle was suspended by a quartz fiber of 9 μ diameter (maker's figure), having a period of swing of about 18 seconds, and a sensibility when charged as above of about 33 cm per volt, as read by a telescope placed with the scale at 1.80 m distance.

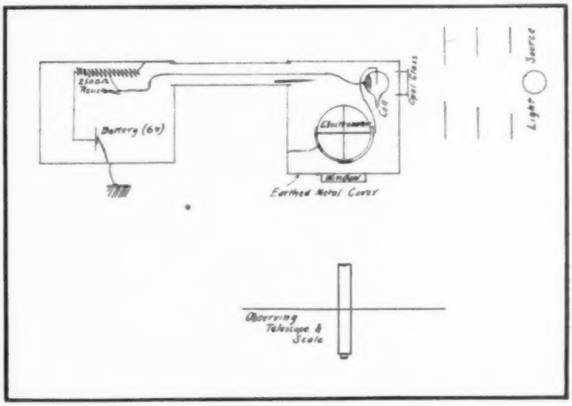


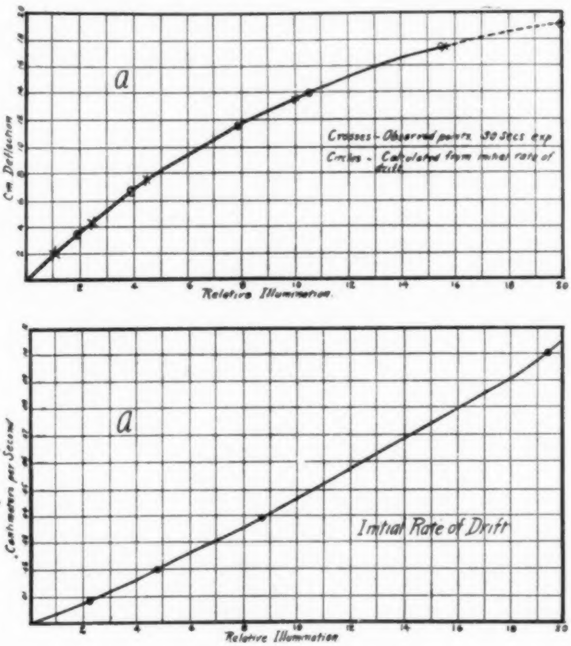
FIG. 1.—Arrangement of apparatus for "rate of drift" and "ballistic" methods of reading photo-electric current.

A standard carbon incandescent lamp of 10 candle-power mounted on a long track served as the light-source. Its light fell upon a piece of flashed opal glass covering the opening to the cell.

With the apparatus so arranged, following the procedure as outlined by Richtmyer, the first experiments made were on the response to various illuminations.

The rate-of-drift method was first used. Contrary to the findings of others, the needle did not "move at a uniform rate," but continuously and rapidly decreased in speed. All tests of insulation and other ordinarily suspected causes of trouble were negative.

Suspecting that a critical condition might exist, caused by the period of the needle having an unusual relationship to the rate of drift, attention was next turned to the second method of measurement above—the ballistic one. In this the cell is exposed for a fixed convenient length of time, the needle allowed to come to rest, and the deflection read.



FIGS. 2 AND 3.—Illumination-current relationships obtained from cell *a* with preliminary apparatus.

Again an unexpected result was obtained, namely, that with each different exposure-time a different illumination-current relationship was found. For short exposure the plotted curve was convex toward the illumination axis. For long exposure it was concave. Fig. 2 shows the curve obtained for 30 seconds exposure. Fig. 3 shows the curve obtained for zero exposure—in other words, the initial rate of drift, as extrapolated from the rate of drifting over successive centimeter divisions on the scale. This latter,

convex to the illumination axis, is of the character obtained by Griffith.

By proper choice, then, of time of exposure, it appeared possible to obtain any curve desired, among others a straight line. But this apparent dependence on time of exposure called for explanation. It was consequently decided to make a trial of the third or steady deflection method, in which the effect of both needle period and choice of exposure-time are eliminated. This led to the construction of the apparatus as finally used, which will now be described, not as chronologically developed, but under appropriate headings.

4. FINAL FORM OF APPARATUS

The electrometer.—After much trouble with defective insulation in damp weather and difficulty in making adjustments, each

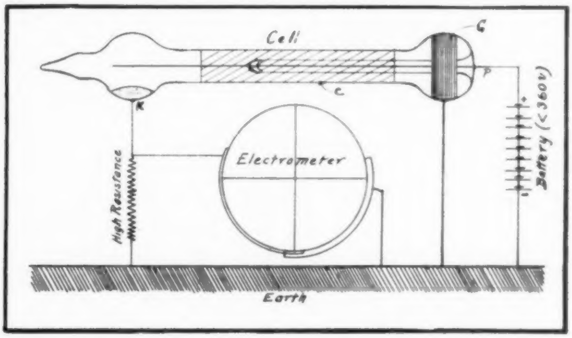


FIG. 4.—Diagram of connections for "steady deflection" method of measuring photo-electric current.

necessitating the removal of the sheet-iron cover, it was decided to inclose the electrometer completely in a dry air-tight box and arrange to operate its adjusting screws from without. Fig. 4 shows diagrammatically the arrangement for the steady deflection method, while Fig. 4*a* shows in section the electrometer and its accessories as practically arranged. The lower portion of the inclosing box is of heavily shellacked wood, lined with tin foil. Around the top is a narrow trough containing mercury. Into this trough sets the sheet-iron top, which in turn has an opening through which the

needle support of the electrometer projects into a cut-off glass bottle, also seated in a mercury trough. By this latter means the electrometer mirror may be turned to bring any part of the scale into view. The leveling screws each rest on a small brass pillar, which moves as a piston in a sleeve mounted in the supporting shelf, stopcock grease making the piston air-tight. Each piston is moved up and down by a tapering rod, which is threaded into a

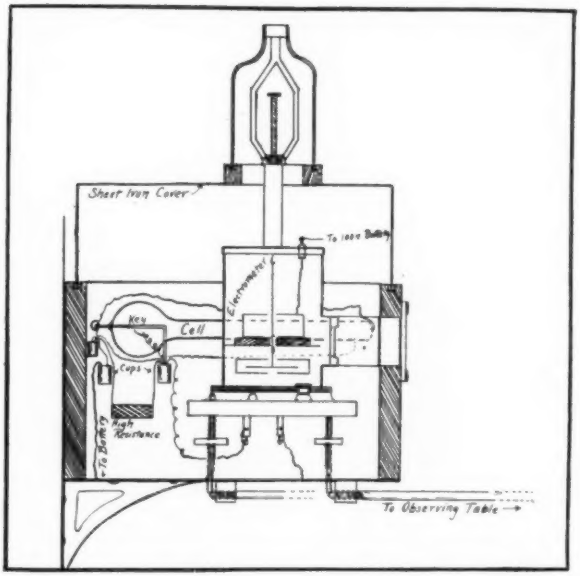


FIG. 4*a*.—Diagrammatic section of final arrangement of apparatus

fixed nut. Long handles carry this adjustment over to the observation table, two meters away. Adjustment is made by charging and discharging the needle with all quadrants earthed and altering the level until no change of needle zero occurs. When adjustment is apparently complete it may be found that the needle does not hang symmetrically with respect to the quadrants. It is then raised or lowered slightly until on adjustment it is symmetrical. The raising or lowering need only be done on first setting up. Slight changes in level are found necessary from time to time as shown by the disturbance of the zero on charging. The adjustment

of the electrometer is a delicate matter and without some means of working from the position of observation is almost prohibitively tedious. When adjustment was completed the deflections were found strictly proportional to voltage over the whole range of the scale.

The needle was charged with the aid of a small ionization chamber containing a small sample of polonium, as described by Erikson.[1] The arrangement of cell, screen, etc., is sufficiently well shown in the diagram. A single key, working through a glass-ground joint, served to earth the electrometer quadrants attached to the cell. Three mercury cups on sealing-wax stands were connected respectively with the source of high potential, the electrometer, and the earthed walls of the box. A metal tube leading down from the box carried the wires to needle and cell from the high-voltage batteries. Wires soldered to the electrometer shelf, cover, and tubes, and to gas pipes insured complete earth connection.

Two trays of calcium chloride rested on top of the wooden part of the box and, in addition, a small tray of phosphorous pentoxide assisted in keeping the inclosure dry. During the later and more important part of the work the basement was steam-heated, whereby everything was made thoroughly dry, even without the drying material. After the cell and electrometer are closed up, they need not be touched for days or weeks, the best possible conditions being thus insured.

Source of high potential.—A battery of small dry cells provided the high voltage necessary to work satisfactorily with the steady-deflection method. These were cells of the kind used in electric flash-lamps, coming in cartons of five each. To insure perfect contact they were taken out of their cases and connected by soldered wires. They were then arranged in rows in four shellacked wooden trays, sixty to the tray. A switchboard on top of the tray-holder was arranged so that one, two, three, or four groups could be put in series, giving, when fresh, 90, 180, 270, or 360 volts. By direct wire connection from the cells intermediate voltages were available when desired. Each tray was connected by fine fuse wires, and a fixed resistance of 2000 ohms was kept in series with the photo-

[1] *Physical Review*, **36**, 253, 1913.

electric cells for their protection. A key, worked by a cord, over a pulley, effected the charging and discharging of the electrometer needle from 65 of the dry cells (100 volts). The entire battery system was inclosed in a galvanized-iron box.

High resistance.—Much time was spent in the search for a satisfactory high resistance. Alcohol in a capillary tube,[1] xylol and alcohol,[2] mannite solution with non-polarizable electrodes,[3] various forms of carbon resistances,[4] besides selenium, Welsbach mantle oxides, etc., were investigated. Alcohol and xylol-alcohol were found to become polarized and were, therefore, abandoned. Mannite solution in a long thermometer capillary proved free from polarization, but had to be made of very low concentration to secure high enough resistance in the length of the tube which it was feasible to use (perhaps owing to the conductivity of the water used as solvent). Its chief defect, however, was the difficulty of preventing evaporation and leakage in spite of the plentiful use of paraffin over all cocks and joints. The most satisfactory resistance was a modification of a carbon one described by Stewart.[5] A piece of dull-surfaced hard rubber had two machine screws tapped into it. Each machine screw carried a nut which could be screwed tightly down against the rubber. Lamp black was mixed in a commercial lacquer and painted around the electrodes, forming an adherent conducting coat. The two spots of lamp black were then joined by a fine lead-pencil line. In this way the chief difficulty with carbon resistance—erratic contact—was overcome. The resistance used in obtaining the curves here shown had a value of 150 megohms. With the electrometer sensibility used, one centimeter deflection thus corresponded to 0.2×10^{-9} amperes.

Source of light.—Electric incandescent lamps were used uniformly, of various candle-powers, their light usually falling upon a piece of flashed opal glass 5 cm from the cell surface. Occasionally when an insensitive cell was experimented with, the opal glass was

[1] Nichols and Merritt, *Physical Review*, **34**, 475, 1912.

[2] Campbell, *Philosophical Magazine*, **22**, 301, 1911; **24**, 668, 1912.

[3] Pohl and Pringsheim, *Berichte d. Deutsch. Phys. Ges.*, **6**, 174, 1913.

[4] Aust, *Physical Review*, **32**, 256, among others, 1911.

[5] *Physical Review*, **26**, 302, 1908.

removed, allowing the light to fall directly on the sensitive surface. The highest illumination used (Figs. 8–15) was about 500 meter-candles on the opal glass, and the glass had an effective absorption of at least 90 per cent as used.

The electric lamps were controlled from storage batteries in the usual way and were mounted upon regular photometer carriages and tracks carrying screens to exclude all stray light. Exposure of the cell to the light was made by a shutter moved from the observing table.

5. CONSTRUCTION AND MANNER OF CONNECTING THE PHOTO-ELECTRIC CELLS

The cell first used was, as above stated, a purchased one, and is shown in section in Fig. 6, *a*. In order to make clear why this cell was not entirely satisfactory and why radical changes in construction were introduced, the manner of connecting up the apparatus will now be given. Fig. 4 shows diagrammatically the essential parts. The alkali metal electrode K is connected to one pair of quadrants of the electrometer and to earth through the high resistance. The other electrode P is connected to the positive terminal of the batteries.

Insulating and guard rings.—Two spurious currents, present in the dark, are found in the photo-electric cell as heretofore constructed. The first of these is opposite in direction to the current produced by light, and is ascribable to the contact difference of potential between the potassium and the other (platinum) electrode P. The second is what has been called a "dark current," in the same direction as the light current. These two currents may be so large as to be very troublesome, especially as they are likely to be variable in amount. No satisfactory results can be secured unless they are reduced to negligible values.

A series of experiments was carried out to learn the cause and method of obviating these currents. A recent paper by Elster and Geitel, appearing since the final form of cell here adopted was under trial, contains all the essential points about these effects, so that it suffices here to say that the "dark current" is the result of conduction over the surface of the glass, due either to the glass

itself, to occluded water vapor, or to a thin film of carbon, deposited from the hydrocarbon with which the alkali metal is usually covered before use. It, in common with the contact e.m.f. current, can be greatly reduced by using glass of good insulating quality, or by separating the two electrodes as far as practicable. A still more complete protection is afforded by the use of internal and external guard rings put on by chemical silvering and connected to earth. The cell shown in Fig. 4 represents the best form. At *G* are the internal and external guard rings, *C* represents an insert of cobalt glass tubing, which has a very high resistance compared with the ordinary clear soda-lime glass. The whole cell is mounted on a

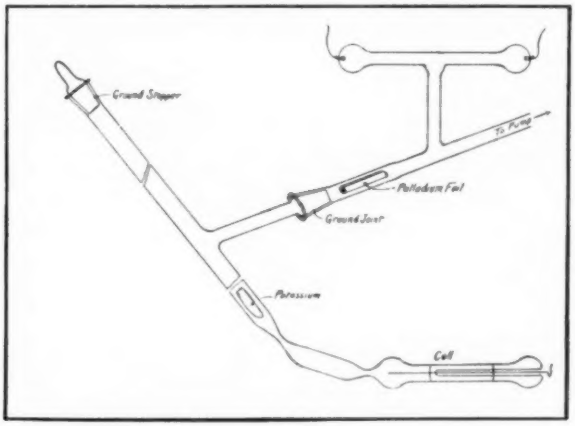


FIG. 5.—Arrangement for filling and exhausting cells

glass plate by a sealing-wax support at *G*, as shown in the sketches of Fig. 6. The insulating and guarding of the electrometer from *P* and from earth connection other than *R* and *G* is, therefore, excellent, as was shown by the fact that a cell similar to *C*, but made of ordinary clear white glass, gave with a certain high resistance and voltage a current due to contact e.m.f. represented by 7 cm deflection, whereas with the cobalt glass inserted, as in *C*, this was reduced to less than a millimeter. The guard ring and cobalt glass together entirely eliminate the "dark current."

Details of filling.—The process of making and filling the cells was not found to be entirely easy, despite the published descriptions

of the process. The pouring of liquid potassium through constricted tubing is very different from pouring mercury, although the two liquid metals look so much alike. Potassium has a very great surface tension, combined with a pronounced tendency to stick to the glass. As a consequence it is likely to pile up in front of a constriction or, on going through, to leave a thread of metal which can be removed only by heating to the distillation point, a

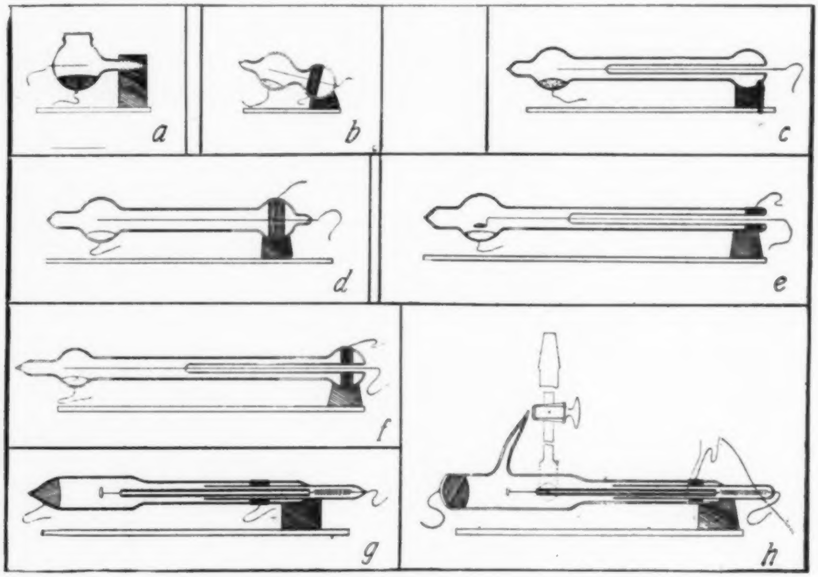


FIG. 6.—Types of cells used in the investigation

process dangerous to the glass. Another difficulty is that of maintaining the surface clean, for, in spite of the most elaborate cleaning, first with hot chromic acid and later with nitric acid, caustic potash, and distilled water as a preliminary to the silvering operation, the glass surface gives up impurities which collect and float on the molten metal. As a result of a great deal of experiment it was found that a large body of potassium could be obtained clean if it was practically shot to place through the filtering constrictions and then allowed to cool at once. Any flowing about caused the collection of a film of scum. In Fig. 5 is shown the essential part

of the apparatus for filling cells. The long tube has at one end a ground stopper through which is introduced a piece of potassium which has been scraped clean under benzol or has previously been distilled into a short glass tube. The Gaede mercury pump is then operated, the cell being inclosed in an electric oven raised to 200° C. Exhaustion is continued for about an hour to allow all the gases to escape from the cell. Then a small flame is played under the metal and glass tube. The potassium presently melts, breaks through its coating of oxide, and, because of the steep slope of the tube, rushes through the two constrictions and into *C*. If necessary, the residue in the lower constriction is driven away by heating. After cooling and then renewed pumping the cell is sealed off. When desired, a small amount of hydrogen is introduced by heating palladium foil in the side tube. Pressures are read by a Gaede short McLeod gauge.

Far easier and more satisfactory is the method of filling by distillation. In this case the unfilled cell is silvered on the lower half of the bulb containing the electrode. The filling tube is inclined at a much less steep angle. A much smaller piece of potassium is needed. A flame is played carefully on the potassium until it vaporizes and the condensing film is driven over and down on to the silver. In order to obtain deposits of uniform character, the expedient was adopted of seating the bulb in a cup of mercury cooled by a jacket of carbon dioxide snow. By slow distillation a perfectly matt fine-grained layer is obtained. If done very slowly on to a highly chilled surface, colored colloidal films may be made.

Description of various cells used.—By the time the serious work on the illumination-current relation was begun, quite a stock of experimental cells was on hand, representing various stages of design. A number of the later ones were pressed into service, and as their individual characteristics probably affect the results obtained with them, a brief description is here given of all the cells for which performance-curves are shown below. Pressures where mentioned are those of the pump system when the cell was sealed off. The gases given off by the molten constriction probably made the actual pressures greater. The cells are shown diagrammatically in Fig. 6.

a) Cell purchased from Müller-Uri. Solid mass of potassium; electrode distance 10 mm; originally possessed no guard rings, but later given an external ring of silvering to be connected to earth; pressure unknown.

b) Short double bulb cell, furnished with inside and outside guard rings; potassium distilled on silver; shortest distance electrode to potassium 10 mm; sealing-off pressure about 0.03 mm.

c) Cell with cobalt glass insert, but no guard ring; potassium distilled; shortest electrode distance 12 mm; pressure not recorded.

d) Cobalt glass insert; guard rings; distilled metal; electrode distance 14 mm; pressure unrecorded.

e) Cobalt insert; guard rings; distilled metal; ring electrode close to metal, distance 8 mm; pressure 0.002 mm.

f) Cobalt glass insert; guard rings; distilled metal, transformed to hydride by glow discharge; electrode distance 15 mm; pressure 0.3 mm.

g) Cobalt glass insert; guard rings; solid mass metal; electrode distance made variable; pressure 0.005 mm.

h) Cobalt insert; guard rings; solid mass of potassium; electrode distance variable; side tube with stop-cock for attachment to pump to permit variation of pressure.

Tubes *g* and *h* will be described more fully under section 9.

Tubes *j* and *k* were kindly loaned the writer by Mr. Saul Dushman, of the General Electric Co. Research Laboratory, near the conclusion of the work. *j* is a cell purchased abroad recently; it has a colored hydride surface, is furnished with exterior tin foil guard rings, and is presumably one of the recent argon-filled type. *k* is a cell prepared by Mr. Dushman, similar in appearance to *a*, but several times larger and without the quartz window. It was exhausted to the best vacuum attainable with a Toepler pump.

6. THE USE OF THE QUADRANT ELECTROMETER FOR THE MEASUREMENT OF PHOTO-ELECTRIC CURRENT

It is unfortunate that students of the photo-electric effect have usually been interested either exclusively in determining currents or exclusively in determining the potential acquired by the sensitive surface. Had they been studying both, there would probably

be fewer unqualified users of the rate of drift method of measuring current, working on the assumption that the electrometer needle moves at a uniform rate indefinitely. As will be shown below, this is true only to a degree of approximation conditioned by the sensibility of the instruments, the effective voltage, and the character of the current. This would seem to be obvious, but the writer has not seen a satisfactory discussion of the matter in any text or article.

The most important fact to keep in mind when using the electrometer to measure current is that the instrument forms part of the electrical system and as such may exert on the phenomenon under study an effect far from negligible. It is imperative that the possible disturbances due to the instrument be thoroughly understood before conclusions are drawn from its indications.

The most general case to consider is the steady deflection method, for it goes over into the rate of drift when the leak becomes zero.

Consider, first, the measurement of a current which is obeying Ohm's law—current proportional to difference of potential.

Let V = the voltage applied to the whole system.
v = potential of the electrometer at any time, t.
R = the resistance, such as a photo-electric cell, through which passes the current to be measured.
r = resistance of the high-resistance leak.

Then, the quantities being represented in such units that the constants of proportionality are all unity, we have for the rate of charging of the electrometer:

$$\frac{dv}{dt}=\frac{V-v}{R}-\frac{v}{r}. \tag{1}$$

Integrating,

$$\frac{V}{R}-v\left(\frac{1}{R}+\frac{1}{r}\right)=ke^{-\left(\frac{1}{R}+\frac{1}{r}\right)t};$$

when

$$t=0,\quad v=0,$$

whence

$$k=\frac{V}{R}$$

and

$$v=\frac{V}{R}\left(\frac{rR}{R+r}\right)\left(1-e^{-\left(\frac{1}{R}+\frac{1}{r}\right)t}\right). \tag{2}$$

When t becomes ∞; i.e., steady deflection,

$$v=V\left(\frac{r}{R+r}\right). \tag{3}$$

Writing this $v=\left(\frac{V}{R+r}\right)r$, it is seen that the deflection v is proportional to the current $\frac{V}{R+r}$ through the whole system, and it follows that if r is made small with respect to R, v will represent the current $\frac{V}{R}$ to any desired degree of approximation. The factor of uncertainty is then the effect of the high-resistance leak. If it is large, then any change in R (such as exposure to light with a photo-electric cell) is not represented by a proportional change in v.

A measure of the relative size of r and R is afforded by the size of the deflection. Let equation (3) be rewritten:

$$\frac{v}{V}=\frac{r}{R+r}.$$

Now $\frac{r}{R+r}$ differs from $\frac{r}{R}$ by an amount immediately calculable for various values of $\frac{r}{R}$. Let us choose arbitrarily the permissible deviation as determined by the experimental accuracy attainable and call this $\triangle\left(\frac{r}{R}\right)$. Find that value of $\frac{r}{R+r}$ for which

$$\frac{r}{R+r}=\frac{r}{R}-\triangle\left(\frac{r}{R}\right).$$

This value is at once the value of $\frac{v}{V}$; in other words, the ratio of the electrometer voltage to the total applied voltage may be used as the criterion of the influence of the high-resistance leak. Taking, for instance, 1 per cent as the permissible error, we have

$$\frac{r}{R+r}=0.99\left(\frac{r}{R}\right)$$

from which

$$R = 99r$$

$$\frac{r}{R+r} = 0.01 = \frac{v}{V}.$$

Consequently, if the current obeys Ohm's law, the largest safe deflection is that corresponding to 1 per cent of the applied voltage.

If, now, the current, instead of obeying Ohm's law, is a saturated one, it is easy to see that the criterion for safety is that the applied voltage shall be sufficiently above the saturation voltage, so that the voltage difference between the electrometer and the source of high potential remains a saturation voltage for all voltages acquired by the electrometer. The difference possible between working with an ohmic current and a saturation current is illustrated in the hypothetical case where 100 volts are applied. If the current obeys Ohm's law one volt deflection is the permissible limit. If the saturation voltage is below 100, say 80, then the electrometer may charge up to 20 volts without the current being misread, i.e., the high-resistance leak may be of 20 times larger value and the sensibility consequently 20 times higher.

Where the current is not clearly of either of the foregoing characteristic types, it is perhaps best to make an experimental determination of the effect of the high resistance. In the present work two resistances of relative strength about 1 to 5 were kept on hand, and in doubtful cases the ratio of large and small illumination-currents was taken with both resistances. Thus in one case where an error of this sort would have vitally affected conclusions, two illuminations gave deflections in the ratio of 3.127 with the higher resistance, and 3.129 with the lower, showing the value of r to be negligibly small.

Turning, now, to the rate of drift and ballistic methods, we obtain the limiting conditions by making $r = \infty$ in equations (1) and (2):

$$\frac{dv}{dt} = \frac{V-v}{R}. \tag{4}$$

$$v = V\left(1 - e^{-\frac{t}{R}}\right). \tag{5}$$

It is evident from equation (4) that the rate of drift of the electrometer needle will be proportional to $\frac{V}{R}$ only if v is negligibly small compared to V. The limiting deflection may be obtained by solving (5) for v for different values of $\frac{t}{R}$, and determining when $\frac{V-v}{R}$ differs from $\frac{V}{R}$ by $\triangle\left(\frac{V}{R}\right)$ in a similar manner to the case just considered. Expressing this in terms of $\frac{v}{V}$ gives the limiting deflection for a given assumed allowable error. For the ballistic method the procedure is exactly similar, the value of $\frac{v}{V}$ being determined for which $\left(1-e^{-\frac{t}{R}}\right)$ differs from $\frac{t}{R}$ by the permissible amount. Allowing 1 per cent as above, this calculation gives $\frac{v}{V}=0.02$ as the limiting deflection.

In case of saturation, it is again obvious that the voltage applied to the system should be large enough so that the difference between the electrometer voltage and the applied voltage is still a saturation voltage.

It is evident from this discussion that the common manner of use of the electrometer for current measurement is valid only if the relation of sensibility to applied voltage is such that the largest charge acquired by the electrometer leaves practically unaffected the effective voltage over the cell or other device. Neglect of this precaution leads to erroneous results, as is pointed out in the next section.

One point worth noting in passing is that with a straight ohmic resistance, it follows from equation (5) that relative current measurements can be made, notwithstanding the non-linear type of the curve connecting voltage and resistance change (current). This is done by determining the *relative times necessary to attain the same deflection;* which follows because v is the same function of t and of $\frac{1}{R}$. Let this be clearly distinguished from determining the *deflection attained in a given time*, which is represented by equa-

tion (5). Both these measures of current are found used indiscriminately by investigators of the photo-electric effect.

7. APPLICATION OF ELECTROMETER DISCUSSION TO PRELIMINARY RESULTS

The application of the preceding discussion makes possible an explanation of the results obtained with the preliminary apparatus.

In Fig. 2 the largest deflection is 19 cm. The sensibility was about $23 \frac{\text{cm}}{\text{volt}}$. Now the potential acquired by a potassium surface under the light of the visible spectrum is not much more than one

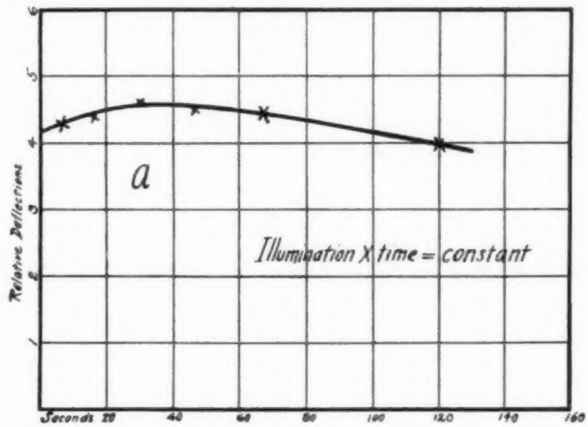


FIG. 7

volt. Substituting the value of 1 for V in equation (5), it is evident that $\frac{19}{23}$ volts is very far from the 0.02 volts determined on as the limiting condition. The curve shown is, therefore, exactly of the type one should expect from an approximately linear illumination-current relationship.

Further points of interest also follow. On the supposition of the cell behaving as an ohmic resistance, it follows that if the current $\frac{1}{R}$ is proportional to the illumination, then the same deflection should be obtained for the same values of the product intensity $\times$ time. This assumption has been made in using the ballistic method, in order to keep the deflection on the scale. Fig. 7 shows

the values of the deflections obtained for constant intensity$\times$time. The deflections are not constant, which would lead to the suspicion that the photo-electric current is not proportional to the intensity. The deviation might, perhaps, be due to a slight leak, for by equation (2) it is $\left(\frac{1}{R}+\frac{1}{r}\right)$ and not $\frac{1}{R}$ that is interchangeable with t. By the use of the steady-deflection method any slight leak merely adds to the purposely introduced leak and hence this alternative explanation may be tested out by recourse to that method.

Now, as to the curve shown in Fig. 3, given by the initial rate of drift. This and all the other results on this cell are consistently explainable on the ground that (1) the true illumination-current relationship is of the character shown in Fig. 3; (2) the effective voltage in the cell is altogether too low to warrant the use of the ballistic method with the electrometer-sensibility employed.

Assuming the current proportional to the voltage, as it probably is to a first approximation, we may substitute in equation (5) above, choosing suitable constants, the current values as given by the initial rate of drift (Fig. 3). The equation $\frac{d}{2}=(1-e^{-25I})$, where d=deflection in centimeters and I=current values from Fig. 3, yields the points marked in Fig. 2 by circles. It is evident from the perfect coincidence that there is nothing thus far to contradict the belief that the curve convex to the illumination axis (as found by Griffith) represents the true illumination-current relationship in this cell.

Further support for this is found in the next section.

8. COMPARATIVE RESULTS ON SEVERAL CELLS

With the apparatus as described under section 4, a large number of miscellaneous illumination-current curves were taken for all the cells above described. These were made with different applied voltages. At first the aim was to use one voltage throughout, and one was selected which would not cause a dark discharge in any of the cells. Afterward it became evident that the phenomenon under study was a function of voltage, so some extra curves were made at different voltages. The results are exhibited in Figs. 8–15.

Before discussing them as a group it is perhaps best to complete the discussion of cell *a*.

At various times during the experiments on high resistances, curves similar to Fig. 3 were obtained from this cell. As long, however, as polarization and other troubles were not entirely eliminated these results could not be accepted as conclusive. Fig. 8 shows the illumination-current curve obtained under conditions believed to be subject to no reservation or correction. It is, like

FIG. 8 FIG. 9 FIG. 10 FIG. 12

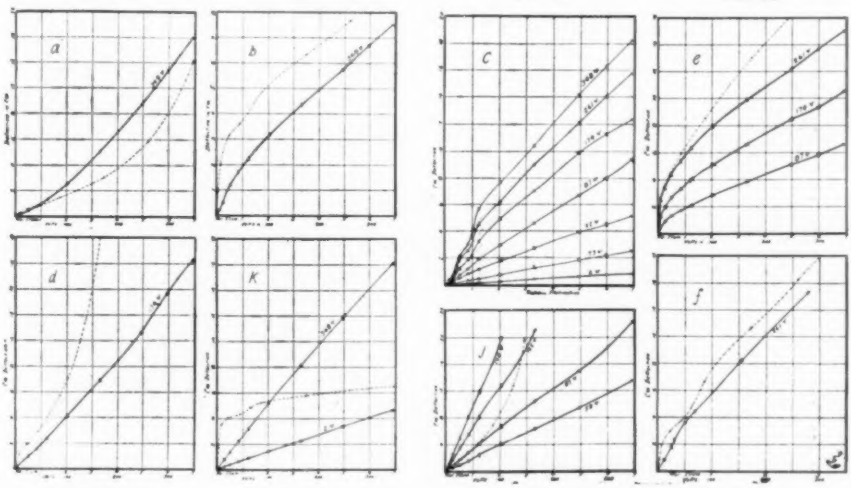


FIG. 11 FIG. 15 FIG. 14 FIG. 13

FIGS. 8, 9, 10, 12, 11, 15, 14, 13.—Characteristics of eight photo-electric cells

Full lines: Illumination-current relationship
Broken lines: Voltage-current relationship
Illumination unit, roughly 10 meter-candles, 1 cm deflection $=0.2\times10^{-9}$ amperes

Fig. 3, the curve given by Griffith, and Lenard's values when plotted, convex toward the illumination axis.

Had no other cell been at hand the conclusion could well have been drawn that this is the true relationship between photoelectric current and illumination. The opportunity afforded by the possession of several cells, however, made it possible to demonstrate that the phenomenon is much more complicated. Each of the several cells, as is evident from Figs. 8–15, shows a different

illumination-current relationship. Curves both concave and convex, of most extreme type, as well as some with double curvature, appear. It appears, too, that the relationship is, to a varying degree, in different cells, a function of the applied voltage.

In no case is this relationship linear. If only a comparatively short range of illumination be used, few points taken, and a certain voltage range not overstepped (see, for instance, *c*, 7.3 volts), curves may be obtained which appear to be straight lines. When compared with other curves of the same family, that is, obtained with higher and lower voltages, the points which deviate by amounts apparently within the errors of measurement are actually found to be true indications of curvature. Furthermore, with the cells *j* and *k*, in which the nearest apparent approach to linearity was found on first measurement, the observations were repeated many times, running up and down the curves, proving the curvatures to be real and of significance. The four curves of cell *j* are obviously developments of each other. Cells such as *j* and k might easily be hastily assumed to show the heretofore believed linear relationship. Only the refinement of measurement called for by the requirements of photometric application would make clear with these particular cells that this is not so. Fortunately, the extreme differences among all the cells leave no doubt of the exceptional and accidental character of the linear relationship when found.

9. SUGGESTED REASONS FOR THESE RESULTS

A study of the illumination-current relationship exhibited in Figs. 8–15 shows that in any one cell the relationship is a function of the applied voltage, although to a varying degree in different cells. Thus the variation in character of curve in cell *c*, in going from 2 to 348 volts, is much greater than in cell *k*. It is evident that different voltages correspond in different cells to different characteristic curves. Thus cells *b*, *f*, and *c* show the same type of curves at voltages 348, 261, and about 100. Nor does it appear possible in any one cell to obtain all types of curves by mere voltage change. For instance, the two extremes exhibited in cell *a* (concave upward) and cell *e* (concave downward) are not attained in cell *c* at the extreme voltages used, other phenomena entering to

interfere at the high voltage and lack of sensibility preventing a study of lower voltage effects.

Differences of pressure, of gas, of electrode distance, of surface, exist among the cells, and it is to these that we must look for explanation of the phenomena. These differences have been recorded in the description of the cells. Additional evidence as to the different electrical conditions holding in the cells is afforded by the voltage-current curves (made subsequently to the illumination-current curves) for a chosen medium illumination. These show differences, ascribable to different pressures and electrode distances, which are as extreme as those between the illumination-curves, which they resemble in some ways.

A study of previous work on the photo-electric effect[1] shows that curves possessing nearly all the characteristics of Figs. 8–15 have been obtained where the variables were voltage, electrode distance, and pressure, illumination being maintained constant. J. J. Thompson[2] develops two equations, the first dealing with the conditions well below the voltage, pressure, and electrode distance at which discharge occurs in the dark and the second with the conditions near the point of discharge. From the first of these equations it follows that for low voltages the current should obey Ohm's law, for higher voltage saturation would set in. From the second of these equations it follows that the current should increase, owing to ionization by collision, according to a power of e determined by electrode distance, etc. These equations were found represented in a general way by replotting the data of cell c, Fig. 10, in terms of voltage and current for fixed illumination, Fig. 16. It will be seen that curves showing various stages of the two equations are represented. It must be clearly understood, however, that the difference between any two curves in this figure is not produced by change of pressure, electrode distance, or gas, but by changing that factor which figures as a constant in Thompson's equations—illumination.

As the most inclusive summary of these results it may be said that the reasoning and equations just quoted are applicable to the

[1] See J. J. Thompson, *Conduction of Electricity through Gases.*

[2] *Ibid.*

present case if variation of illumination and variation of voltage are considered as similar in influence. But certain peculiarities of these illumination-curves are not to be overlooked. The voltage-current curves of cell *c* show at the lowest illumination apparent complete saturation; at slightly higher illuminations approach to saturation and subsequent increase ascribed to ionization by collision. But at higher illuminations the increased current again

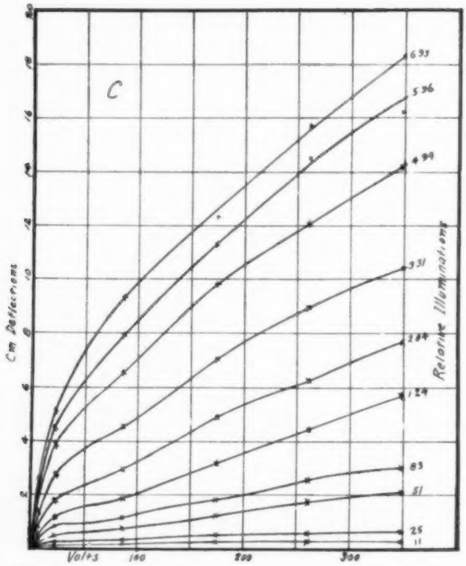


FIG. 16.—Voltage-current relations in cell *c* at different illuminations

approaches saturation, making a saturation-curve with a depression. At still higher illuminations this depression disappears and a simple characteristic curve is obtained, apparently approaching saturation uniformly. This steplike character of the voltage-current curve occurs in a number of cells, as shown by broken lines in Figs. 8–15. These data are of interest in connection with the characteristic curves for the inert gases, recently published by Franck and Hertz,[1] which show a series of steps.

The most striking feature of the illumination-current curve *c* is

[1] *Berichte der Deutsch. Phys. Gesel.*, **20**, 929, 1913.

the series of steps shown at higher voltages. These steps, caused by varying illumination, are similar to the voltage-current curve steps just referred to, and are additional evidence that illumination variation must in these cells be considered as of similar effect to voltage variation. The data of cell *j* (argon-filled) indicate similar stepped curves, due to illumination variation. A peculiarity of curve *c* is indicated by the broken-line continuations of the first approach to saturation. In each case the point of illumination 0.8 was the second point of rest of the electrometer needle, attained after the needle had nearly come to rest at a lower value. Undercooling-curves are at once suggested.

The next step in the study appeared to be the intentional variation of electrode distance and pressure, the factors not variable in the completed cell.

10. FURTHER EXPERIMENTS TO TEST CONCLUSIONS

Cell with variable electrode distance.—The easiest factor to vary and, as it was thought (noting the results of Stoletow), the most likely to show in the results, was electrode distance. A special cell was, therefore, constructed, shown in Fig. 6 (*g*). The platinum electrode was attached to an iron rod which slid in a glass tube and was connected by a fine coil of copper wire to a platinum wire sealed in the glass and going to the batteries. The cell was first filled in a horizontal position and sealed off at the best vacuum obtainable with the Gaede mercury pump, the iron rod was then held in position by a solenoid, the cell turned to the upright position, and the potassium melted and flowed into its final place. The electrode was afterward easily put in any desired position by inclining and tapping the tube.

Fig. 17 shows two curves obtained with the cell, for electrode distance of 2 and 40 mm. These are both concave to the illumination axis, and while the short-distance curve is somewhat less concave than the other, it is evident that variation of electrode distance alone, at this pressure, does not produce rapid changes in curve type.

Cell with variable pressure.—A cell similar to *g* was next constructed, differing in the possession of a side tube containing a

ground stopcock and a ground cone, fitting the ground sleeves of the pump system *h*. This cell was made a little differently from the last, being filled in an upright position through a lateral constricted tube, the solenoid being in position throughout. Of course the proper procedure would be to have the cell constantly connected to pump and gauge, but this was not possible to arrange in the present case. Trouble was expected from the stopcock, and leakage did, in fact, spoil the potassium surface after two days. In that

FIG. 17

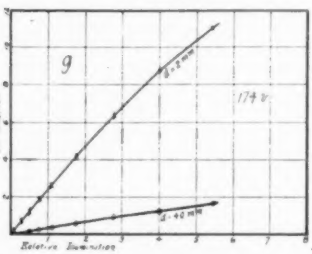
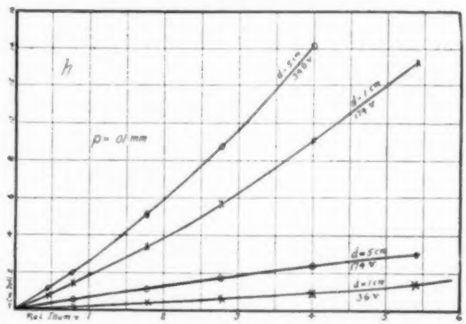


FIG. 18



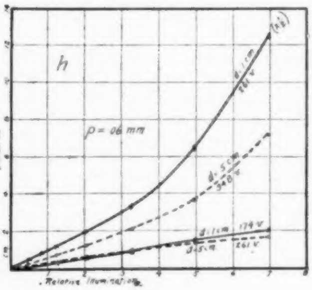


FIG. 19

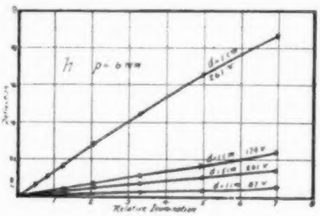


FIG. 20

FIGS. 17, 18, 19, 20.—Illumination-current relationships in cells of variable electrode distance and gas pressure.

time, however, it was possible to allow the surface to reach a steady state after its first rapid drop in sensitiveness and to run several curves at several pressures of hydrogen. The change in pressure was made by replacing the cell on the pump, securing a good vacuum, and introducing gas from palladium foil. Measurements were made as soon after as practicable. Results for three pressures 0.01, 0.06, and 0.6 mm are shown in Figs. 18, 19, and 20, while Fig. 17 may be considered as exhibiting part of the same series of pressures, the

value of p being about 0.005 mm. These curves, while by no means offering a complete picture of the variation of the illumination-current relation, show clearly that it is a function of pressure, voltage, and electrode distance. From the results of Varley it is to be expected also that the nature of the gas filling the tube will affect the relationship.

II. DISCUSSION

Two generalizations appear to be justified from study of these curves:

A. There is a qualitative agreement throughout between the effects of (1) increase of electrode distance; (2) increase of pressure; (3) decrease of voltage. For illustration, take Fig. 19, $p=0.06$ mm, electrode distance 1 cm, 261 volts. This gives a curve convex to the illumination axis. Lowering the voltage to 174, distance remaining constant, yields a nearly straight line, i.e., a more concave curve. Leaving the voltage at 261, an increase of electrode distance to 5 cm yields a concave curve. With this greater electrode distance an increase of voltage to 348 brings back the convex type of curve. Again changing from 174 volts, 1 cm distance at 0.01 mm (Fig. 18), to the same voltage and distance for 0.06 mm (Fig. 19) alters the curve from convex to straight, while with pressure remaining constant at 0.01 mm an increase of electrode distance to 5 cm yields a concave curve. There appears to be no exception over the range of voltages, etc., here used, to this relationship, which, of course, is not an unnatural one.

B. The course of the changes from the most intense conditions (high voltage, low pressure, small electrode distance) to the least intense (low voltage, high pressure, large electrode distance) appears to consist of at least three and perhaps more stages.

First stage.—At most intense conditions (Fig. 15) a current which approaches saturation with increase of illumination (curves concave toward illumination axis).

Second stage.—A current increasing with illumination in the same manner that a constant illumination current increases as the dark discharge voltage is approached (ionization by collision) (Figs. 8, 11, 18, 19).

Third stage.—This second current approaches saturation (Figs. 9, 12, 13).

A fourth stage is suggested in the curves of Fig. 10, cell *c*, where at relative illumination 0.8 for the higher voltages, a new upward turn is taken, followed again by apparent slow approach to saturation. In fact it appears probable that the illumination-current relationship may be plotted as a curve whose ordinates increase in value in steps.

The most general statement of the effects produced by light is that already made, that variations of illumination appear to act in a similar manner to variations in voltage, producing currents obeying Ohm's law over short ranges (in this case $c=\frac{\text{Illumination}}{\text{Resistance}}$), currents which approach saturation, which increase in value as though through ionization by collision, etc. The quantity of electrons liberated is apparently as potent a factor as is voltage in altering the character of the discharge.

Other differences between the originally studied cells may have been of effect. For instance, the somewhat different character of surface with consequent different amounts of normal and selective effect. It would be of interest in a more complete and detailed study of this relation to examine the normal and selective effects separately in sodium potassium alloy.

A point of extreme importance which must here be emphasized is that these results apply of necessity only to gas-filled cells. The best vacuum attained in the cells here described was not sufficient to produce saturation. Traces of gas, and probably mercury vapor, were always present. What will happen in a cell exhausted by the new Gaede molecular pump with the assistance of liquid air is a question still open for investigation. It must be remembered, however, that all previous work on this relationship has been done with no better vacua than here used; that the most widely exploited cells, those of Elster and Geitel, are purposely filled with a gaseous atmosphere. In short, this work stands in contradiction to all researches which have hitherto been considered proof of the linear illumination-current relationship.

Looked at from the standpoint of photometric application,

these results are the opposite of encouraging. The most that can be said is that by careful choice of electrode distance, gas, pressure, and voltage, cells may be produced which for a more or less limited range show a linear relationship between illumination and current.

Every cell would, therefore, have to be either tested for this relation while still on the pump, or have its calibration-curve determined under conditions to be rigidly adhered to afterward. Any change in pressure, or possibly (a point not touched on here) any aging of the surface would necessitate a checking of this calibration.

A further question, should colored-light photometry be attempted with cells made with the characteristic thus determined, would be the uniformity between different cells and the permanence in any one cell of the distribution of sensibility through the spectrum. Data on wave-length sensibility obtained during the course of this investigation indicate possibilities of wide differences from cell to cell, but this question is left for further study.

It must not be forgotten, however, that the photo-electric cell possesses enormous sensitiveness and that the conclusions above reached do not affect at all its use as a detector, or for measurement by substitution methods where the lights under comparison are of identical quality. A set of electric lamps of the same type could, for instance, be brought to the same candle-power by so regulating their voltage that they gave the same photo-electric current in the cells, or relative candle-powers could be determined by finding the distance at which different lamps gave the same deflection. A degree of sensitiveness greatly exceeding the eye should be attainable for such work. There is not the need, however, for this kind of photometric adjunct that exists for an "average eye" for standardization of color measurements.

12. SUMMARY AND CONCLUSIONS

1. Photo-electric cells and auxiliary apparatus have been developed by which the character of the photo-electric current may be studied without disturbance by spurious effects.

2. The use of the quadrant electrometer for measuring photo-electric current has been studied and the conditions determined such that its characteristics introduce no distortions in the results.

3. The illumination-current relationship has been found not to be linear, but to be a complicated function of voltage, electrode distance, and pressure, similar to the voltage-current relationship.

4. It is concluded that gas-filled photo-electric cells do not possess the qualities most desirable in a physical photometer.

The writer takes pleasure in acknowledging the assistance and co-operation of Mr. E. Karrer during a portion of this work.

Physical Research Laboratory
United Gas Improvement Co.
Philadelphia
January 1914

PHOTOMETRIC TESTS OF SPECTROSCOPIC BINARIES

By JOEL STEBBINS

For some years, I have been interested in the possible small light-changes of spectroscopic binaries, synchronous with the orbital motion. In 1904 I began to test some of these stars with a visual photometer, but did not succeed in finding any new variables, and the problem was laid aside until some means could be found of improving the accuracy of the observations. When the selenium photometer was perfected, I at once planned to renew the tests of short-period binaries. It had only been my hope, from the start, to detect continuous variations of something like 0.05 or 0.10 magnitude, and the possibility of finding eclipsing systems had not appealed to me until eclipses had been actually discovered in *β Aurigae* and *δ Orionis*. The success in these two cases brought out the high probability that there are plenty of eclipsing variables which may easily be detected by observations at the proper times. All we need to do is to compute from the spectroscopic elements the instants when the longitude from the node, u, is equal to $90°$ or $270°$. An eclipse of the second component is just as important as that of the primary, for, neglecting the eccentricity of the orbit, where there is one minimum there must always be two. Even if the fainter body is nonluminous, it will reflect the light of the primary, and theoretically at least this second eclipse will be present.

This program of taking photometric measures at the favorable epochs sounds easy enough, but the difficulties are considerable. We hear some complaint that the observers of *Algol* variables do not determine the light-curves thoroughly, but a little experience will convince anyone that, at an ordinary station, it is not often that a critical point on a light-curve, such as the beginning or end of a minimum, happens to come on a good clear night, with the star in proper observing position. In work on bright stars with the selenium photometer, the times are more than ordinarily restricted because of the exacting requirements. When a variable and the

comparison star are 10° or even 15° apart, and we desire results correct to the hundredth of a magnitude, there is absolutely no use in trying to work on any but first-class nights.

Up to the present, with our 12-inch telescope, I have not tried the selenium method on stars fainter than magnitude 3.0, and the program is therefore quite limited. Nevertheless it has seemed worth while to continue the search for variables, because of the importance of those cases where we have both the light- and velocity-curve of an eclipsing system. I have therefore made a list of spectroscopic binaries, and at the beginning of each month I have marked the nights when each star should be tested. The work of observation is slow and laborious, but there is always the hope of picking up a variable, and any such found will be both interesting and important.

The times of minima are easily computed with the aid of the "Tables for the True Anomaly in Elliptic Orbits" by Schlesinger and Miss Udick.[1] When the eccentricity is small, we may derive a convenient check from the relation that the true anomalies of the two minima differ by 180°. Let t_1 be the epoch of eclipse of the brighter, and t_2 that of the fainter spectroscopic component, v_1, v_2, and M_1, M_2 the corresponding true and mean anomalies; P the period, e the orbital eccentricity, and ω the longitude of periastron. Then[2]

$$\left.\begin{aligned} v_1-M_1=2[e\sin v_1-\tfrac{1}{2}\frac{1+2\sqrt{1-e^2}}{(1+\sqrt{1-e^2})^2}e^2\sin 2v_1+ \\ \tfrac{1}{3}\frac{1+3\sqrt{1-e^2}}{(1+\sqrt{1-e^2})^3}e^3\sin 3v_1-\ldots\ldots] \end{aligned}\right\} \quad (1)$$

$$\left.\begin{aligned} v_2-M_2=2[e\sin v_2-\tfrac{1}{2}\frac{1+2\sqrt{1-e^2}}{(1+\sqrt{1-e^2})^2}e^2\sin 2v_2+ \\ \tfrac{1}{3}\frac{1+3\sqrt{1-e^2}}{(1+\sqrt{1-e^2})^3}e^3\sin 3v_2-\ldots\ldots] \end{aligned}\right\} \quad (2)$$

Subtracting (2) from (1) and noting that $v_2=v_1+\pi$, also $v_1=\frac{\pi}{2}-\omega$, we have, neglecting e^5 and higher powers:

$$M_2-M_1-\pi=4e\cos\omega-\tfrac{2}{3}e^3\cos 3\omega$$

[1] *Publications of the Allegheny Observatory*, **2**, 155, 1912.

[2] Tisserand, *Mecanique céleste*, **1**, 224.

or

$$\frac{(M_2-M_1)P}{2\pi}=t_2-t_1=\frac{P}{2}\left(1+\frac{4e\cos\omega}{\pi}-\frac{2}{3\pi}e^3\cos 3\omega\right) \tag{3}$$

This gives a convenient check upon the computation of t_1 and t_2, but cases may arise where we may neglect e^3 and use the times t_1 and t_2 for the determination of e. From (3) there follows

$$e\cos\omega=\frac{\pi}{2}\left(\frac{t_2-t_1}{P}-\frac{1}{2}\right) \tag{4}$$

The determination of the spectroscopic elements is often troublesome when e is small, and if it happens that a star shows eclipses which can be accurately observed, then $e\cos\omega$ may be derived from (4), and only the other component, $e\sin\omega$, need be found from the velocity measures.

Let us note that the times, t_1 and t_2, here defined are not strictly the instants of maximum eclipse when the orbital inclination differs from 90°, but the errors introduced are quite small.

It is the custom for most observers to be reasonably conservative in the announcement of new variable stars, especially those with small range. In the case of stars here considered it is quite legitimate to publish suspicions of variability, because in any case the spectroscopic period may be assumed, and other observers will know just when to confirm the light-changes. Negative evidence is also important, and when a star has been tested without showing eclipses, that fact ought to be announced with reasonable promptness so that a second observer will not waste his time on unpromising cases. In this connection, however, it is much more difficult to prove that the light of a star is constant than that it varies. When eclipses take place, and the star decreases by a conspicuous amount, the fact is settled; but to show that there are no eclipses, one must have observations extending for some time on each side of the predicted minimum, to make sure that the spectroscopic elements and period are all right. It has appeared to some astronomers that the laborious solution for the definitive elements of a spectroscopic binary are scarcely worth while, but I have found it extremely convenient to know that the spectroscopic results are reliable.

In forming the observing program, Campbell's "Second Catalogue of Spectroscopic Binary Stars"[1] has been used, though in some cases extra decimals have been taken from the original sources. The stars have been selected principally on account of brightness. The favorable cases are presumably those systems of short period, and large range in velocity; or, what amounts to the same thing, those with large values of the quantity $\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$. Some stars have been tested on many nights and throughout the whole period, but others only a few times, and there is therefore a vast difference in the relative thoroughness with which the different objects have been studied.

In what follows, under each star the spectroscopic elements are given, and with these the hypothetical times of eclipses. Greenwich Mean Time is used throughout. In the journal of observations, the phase is expressed as a fraction of the period. A "set" of measures usually consists of two exposures on the comparison star, four to six on the tested star, and then two on the comparison star. All of the measures have been corrected for atmospheric absorption. After a discussion of the evidence in each case, I have tried to give in a short summary my best judgment of the result of the tests.

21 α Andromedae

H.R. 15, Mag. 2.15, Spectrum A0

There are independent orbits for this star by Baker[2] and by Ludendorff.[3]

	Baker	Ludendorff
P	$96\overset{d}{.}67$	$96\overset{d}{.}7$
T	2417882.40	2416817.
ω	$76\overset{\circ}{.}21$	$69\overset{\circ}{.}4$
e	0.525	0.50
$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	0.180	0.13

The hypothetical light-elements from Baker's orbit are:

$$\text{Min. I} = \text{J.D. } 2417883.39 + 96\overset{d}{.}67 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 57\overset{d}{.}26 = 0\overset{P}{.}592$$

[1] *Lick Observatory Bulletin*, **6**, 17, 1910.

[2] *Publications of the Allegheny Observatory*, **1**, 22, 1908.

[3] *Astronomische Nachrichten*, **178**, 23, 1908.

The same from Ludendorff are:

$$\text{Min. I} = \text{J.D. } 2416818.6 + 96^{d}.7 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 60^{d}.5 = 0^{P}.626$$

The comparison star was *β Andromedae*, with additional measures on *α Pegasi* for the absorption correction. The difference of magnitude is in the sense: *β Andromedae minus α Andromedae*.

TABLE I

OBSERVATIONS OF *α Andromedae*

Date	G.M.T.	Phase		Difference of Magnitude	Sets	Remarks
		Baker	Ludendorff			
1911		P	P	Mag.		
October 22.....	20^h 17^m	0.994	0.000	0.37	3	
23.....	17 15	.003	.010	.37	3	
23.....	18 13	.003	.010	.36	3	
November 15.....	17 48	.241	.248	.37	2	
December 19.....	15 04	.592	.598	.37	3	Poor sky
22.....	15 00	0.623	0.629	0.32	2	Smoke

On December 19 the sky was somewhat thick, but apparently uniform, and on December 22 smoke was passing near or over the stars. In determining the light-curve of a known variable, I should not think of working under such conditions; but in this case there was a chance to make a discovery, since experience shows the smoke would not change the result by more than 0.1 or 0.2 magnitude. The agreement of the other measures is in part accidental but not extraordinary.

Result for α Andromedae.—Observations on five nights near primary and secondary minimum, and on one night between minima, give no evidence of eclipse variation.

13 α Aurigae

H.R. 1708, Mag. 0.21, Spectrum G0

The orbit is by Reese,[1] and two spectra are visible.

P..............	$104^{d}.022$	$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$..............	0.184
T..............	2414899.5		
ω..............	$117^{\circ}.3$	$m_1 \sin^3 i$..............	1.19
e..............	0.016	$m_2 \sin^3 i$..............	0.94

[1] *Lick Observatory Bulletin*, 1, 34, 1901.

The hypothetical light-elements are:

$$\text{Min. I} = \text{J.D. } 2414995.87 + 104^{\mathrm{d}}.022 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 51^{\mathrm{d}}.52 = 0^{\mathrm{P}}.495$$

A variation of *Capella* seems improbable because of the length of period and the Class G spectrum. After waiting during two years we managed to observe it near minimum on one night but through smoke. The comparison star was the variable *β Aurigae*, which, however, was not near its eclipsing phase. The difference of magnitude is in the sense: *β minus α*.

TABLE II

OBSERVATIONS OF *α Aurigae*

Date	G.M.T.	Phase	Difference of Magnitude	Sets	Remarks
1912		P	Mag.		
March 6	$15^{\mathrm{h}}\ 34^{\mathrm{m}}$	0.998	1.70	4	Smoke
6	16 14	.999	1.79	4	Smoke
6	16 54	.999	1.84	4	Smoke
March 6	16 14	.999	1.78	12	Mean
9	16 29	0.028	1.77	5	

Result for α Aurigae.—Measures on one night at phase $0^{\mathrm{P}}.999$ compared with another night at phase $0^{\mathrm{P}}.028$ give no evidence of variation.

44 ι Orionis

H.R. 1899, Mag. 2.87, Spectrum Oe5

The orbit is by Plaskett and Harper.[1]

P	$29^{\mathrm{d}}.136$	e	0.754
T	2417587.993	$\dfrac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	1.14
ω	$113^{\circ}.28$		

The hypothetical light-elements are:

$$\text{Min. I} = \text{J.D. } 2417587.815 + 29^{\mathrm{d}}.136 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 7^{\mathrm{d}}.129 = 0^{\mathrm{P}}.245$$

Eclipses of this star would be especially interesting because of its spectrum, Oe5. The comparison star was *53 κ Orionis*, and the difference of magnitude is in the sense: *κ minus ι*.

[1] *Astrophysical Journal*, **30**, 379, 1909.

TABLE III

OBSERVATIONS OF *ι Orionis*

Date	G.M.T.	Phase	Difference of Magnitude	Sets	Remarks
1911		P	Mag.		
November 24......	$20^h\ 14^m$	0.025	0.65	2	
December 11......	18 28	.606	.64	2	
1912					
February 19......	14 38	.003	.64	5	Smoke
19......	15 28	.004	.62	5	Smoke
March 26......	14 54	0.239	0.60	3	Bright moon, poor

Result for ι Orionis.—Observations at $0^P.004$ after primary and $0^P.006$ before secondary minimum show no evidence of variation.

66 α Geminorum

α_1 *Geminorum, H.R.* 2890, Mag. 2.85, Spectrum A0

α_2 *Geminorum, H.R.* 2891, Mag. 1.99, Spectrum A0

The orbit for each star has been derived by Curtis.[1]

	α_1	α_2
P................	$2^d.928285$	$9^d.218826$
T................	2416828.057	2416746.385
ω................	$102^\circ.52$	$265^\circ.35$
e................	0.01	0.503
$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$..........	0.0097	0.0015

The hypothetical light-elements are for α_1:

$$\text{Min. I} = \text{J.D. } 2416827.957 + 2^d.928285 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 1^d.460 = 0^P.499$$

and for α_2:

$$\text{Min. I} = \text{J.D. } 2416751.305 + 9^d.218826 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 4^d.333 = 0^P.470$$

I have studied this quadruple system in some detail, principally because of the favorable comparison star, *β Geminorum*. As both the mass functions are small, there seems to be little probability of eclipses, but the high eccentricity for α_2 might lead to some light-variation. The companion in this case is three times nearer and

[1] *Lick Observatory Bulletin*, **4**, 58 and 64, 1906.

receives nine times as much radiation at periastron as at apastron. A variation of 0.02 mag. in the combined light of α_1 and α_2 would be produced by a change of 0.064 mag. in α_1, or 0.029 mag. in α_2. There is, therefore, a possibility that the most accurate observations might detect a variation in either of the stars.

As the work on *Castor* was secondary to that on other stars, the observations were prolonged over several years, and with varying conditions of the photometer. We used different mountings of the selenium cell, and different portions of its sensitive surface. For this reason, the observations are divided into five series, and small systematic corrections have been applied to the results so as to bring the mean of each series into agreement.

Series	Correction Mag.
I	−0.001
II	+ .001
III	.000
IV	+ .010
V	−0.003

Since the maximum difference between series was only 0.013 mag., the experiments with the instrument introduced no great disturbance.

In Table IV are given the results of the measures, after the above systematic corrections have been applied. The weights are proportional to the number of sets, except for Series II, where the general agreement is poor, and the whole series has been given half-weight. The difference of magnitude is in the sense: *α Geminorum minus β Geminorum*, the measure being, of course, of the combined light of α_1 and α_2.

On February 9, 1912, I was surprised to note that the measures made *Castor* abnormally faint at a phase near minimum of α_1. The observations were therefore extended as long as possible at the next minimum, three nights later, and surely enough the light decreased again about 0.05 magnitude. I then looked over the previous selenium observations, and found another discordant result at the same phase. Also a search through some visual work gave two more discordances at the eclipsing phase of α_1. I therefore had

TABLE IV

OBSERVATIONS OF α *Geminorum*

SERIES I

Date		G.M.T.	Phase α_1	Phase α_2	Difference of Magnitude	Weight
1909			P	P	Mag.	
November	25...	$18^h\ 38^m$	0.706	0.524	0.237	5
December	7...	17 47	.792	.822	.217	4
	18...	18 20	.556	.018	.233	5
1910						
January	9...	18 43	.075	.406	.231	5
	18...	19 02	.153	.383	.217	5
	27...	18 33	.219	.358	.205	5
	31...	19 18	.596	.795	.207	5
February	1...	18 16	.923	.899	.227	5
	3...	17 15	.591	.111	.231	5
	3...	18 20	.607	.116	.235	3
	4...	17 48	.940	.222	.197	5
March	12...	16 40	.218	.122	.217	5
	13...	16 44	.561	.231	.210	5
	14...	15 21	.883	.333	.222	5
	16...	15 03	.561	.548	.213	5
	17...	15 10	0.905	0.657	0.223	5

SERIES II

Date		G.M.T.	Phase α_1	Phase α_2	Difference of Magnitude	Weight
April	3...	14 06	0.695	0.497	0.217	2
	6...	14 11	.720	.822	.238	2
	9...	14 16	.746	.148	.217	2
	10...	13 54	.082	.255	.242	2
	10...	14 49	.095	.259	.206	2
	13...	13 58	.108	.581	.199	2
	13...	14 48	.120	.585	.168	2
	13...	16 08	.138	.591	.209	2
	20...	15 29	.520	.347	.251	2
	20...	16 06	0.529	0.350	0.253	2

SERIES III

Date		G.M.T.	Phase α_1	Phase α_2	Difference of Magnitude	Weight
November	24...	19 42	0.026	0.013	0.209	4
December	12...	20 41	.187	.970	.231	5
	12...	21 38	.200	.975	.221	5
1911						
January	9...	16 25	.688	.988	.218	5
	9...	17 15	.700	.992	.246	5
	9...	18 07	.712	.996	.220	5
	9...	19 01	.725	.000	.208	5
	9...	19 52	.737	.004	.203	5
February	24...	17 17	.409	.982	.197	5
	24...	18 10	.422	.986	.189	5
March	4...	17 40	.147	.851	.227	4
	13...	16 41	.206	.823	.222	5
	20...	17 33	0.609	0.587	0.244	5

TABLE IV—*Continued*

SERIES III—*Continued*

Date	G.M.T.	Phase α_1	Phase α_2	Difference of Magnitude	Weight
1911		P	P	Mag.	
March 22...	15^h 45^m	0.266	0.795	0.259	5
24...	15 20	.944	.010	.215	5
27...	15 39	.973	.337	.220	5
31...	16 22	.349	.774	.191	5
April 1...	15 10	.673	.877	.212	5
9...	14 49	.400	.744	.231	5
15...	15 16	.456	.396	.220	5
22...	14 46	.839	.153	.214	5
22...	15 41	.852	.158	.207	5
23...	14 48	.181	.262	.210	5
23...	15 43	.194	.266	.201	5
24...	15 41	.535	.375	.228	5
May 7...	14 34	.959	.780	.240	5
7...	15 26	0.971	0.784	0.248	5
SERIES IV					
November 26...	20 33	0.367	0.827	0.233	5
26...	21 26	.380	.831	.220	5
December 1...	21 21	.086	.373	.237	5
3...	22 13	.782	.594	.240	5
3...	23 22	.798	.599	.221	5
1912					
January 8...	18 48	.027	.483	.224	3
12...	19 40	.405	.921	.196	5
12...	20 59	.424	.927	.199	5
18...	18 55	.444	.569	.230	5
18...	19 49	.457	.573	.220	5
February 9...	20 12	.975	.961	.202	5
9...	21 39	.995	.968	.246	5
12...	18 02	.968	.277	.199	5
12...	18 58	.981	.281	.222	5
12...	19 53	.994	.285	.229	5
March 5...	16 40	.462	.657	.212	5
5...	18 07	.483	.663	.235	5
5...	19 25	.501	.669	.214	5
15...	15 29	.860	.736	.199	5
15...	16 24	0.873	0.740	0.225	5
SERIES V					
March 24...	15 37	0.936	0.713	0.250	5
25...	16 20	.287	.825	.219	5
30...	15 57	.989	.365	.218	4
30...	16 45	.001	.369	.233	4
April 7...	14 42	.703	.228	.209	3
8...	15 16	.053	.339	.224	5
10...	14 38	.727	.553	.227	5
15...	14 20	.430	.094	.189	3
22...	14 29	.823	.854	.220	5
26...	14 38	0.191	0.288	0.211	4

five independent measures all showing that this is an eclipsing variable. On the theory of probabilities, such an agreement could scarcely be the result of chance, but nevertheless there were some suspicious circumstances. Further search revealed one or two good measures which did not confirm the variation, and there was also the possibility that a systematic error had crept in, due to increased atmospheric absorption with large zenith distances, or with a poor sky. Tests for this effect had been taken again and again on other stars, but no measurable variation with zenith distance had been found; this means, of course, any outstanding variation after the usual correction for differential absorption is applied. Here, however, we have the complication that both components of *Castor* are of spectrum Class A, while the comparison star, *Pollux*, is of Class K. It is natural to suppose that the light of the bluer star will suffer greater absorption near the horizon. Also it is possible that the color sensibility of selenium changes with varying light-intensity. Either of these effects would account for the discordances.

The amount of the absorption at any time is indicated by the size of the galvanometer deflections, and a graphical treatment showed clearly the dependence of the difference of magnitude upon the deflections. The matter was further tested by observing under extreme conditions, and in Table V are given the results of these tests. On these dates, the sky was "thick," but apparently uniform. The deflection is that of *Pollux* in terms of the mean of adjacent dates. For instance, the first two deflections of 0.6 and 0.4 were small compared with the remainder of the series, where all other deflections ranged from 0.8 to 1.2, the unit being 30 mm. The most discordant observation, on March 30, 1912, was taken at zenith distance 73°, and with the smallest deflections.

The results in Table V are not very accordant as to the amount of the absorption effect, but a good agreement could not be expected with the character of the sky. The point to emphasize is that a small deflection always corresponds to a positive residual, i.e., *Castor* faint. The measures in Table IV have already been corrected for the absorption effect, where necessary, most of the corrections being only one- or two-hundredths of a magnitude. The test observations of Table V are not used in the further discussion.

The next step is to form normal magnitudes from Table IV on the basis of the phase of a_1. In Table VI each normal is usually from three observations, or the weighted mean of 12–15 sets. With a mean difference of 0.220 mag., the residuals give the probable

TABLE V

TEST OBSERVATIONS OF *a Geminorum*

Date	G.M.T.	Phase a_1	Phase a_2	Difference of Magnitude	Residual	Deflections	Sets
1911		P	P	Mag.	Mag.		
May 2	16^h08^m	0.273	0.244	0.290	+0.070	0.6	5
4	15 57	.954	.461	.344	+ .124	.4	5
1912							
February 12	20 34	.004	.288	.266	+ .046	.6	3
12	21 18	.015	.291	.253	+ .033	.4	3
March 8	19 16	.524	.994	.317	+ .097	.6	5
8	20 13	.537	.998	.314	+ .094	.4	5
30	19 12	.036	.380	.463	+ .243	.35	3
April 5	13 54	.009	.007	.312	+ .092	.7	5
5	15 04	.026	.012	.350	+ .130	.5	4
5	16 26	0.045	0.018	0.421	+0.201	0.4	4

TABLE VI

NORMAL MAGNITUDES, a_1 *Geminorum*

Phase	Difference of Magnitude	Residual	Phase	Difference of Magnitude	Residual
P	Mag.	Mag.	P	Mag.	Mag.
0.038	0.219	−0.001	0.597	0.223	+0.003
.082	.231	+ .011	.657	.225	+ .005
.139	.210	− .010	.702	.232	+ .012
.186	.218	− .002	.719	.218	− .002
.200	.215	− .005	.734	.215	− .005
.234	.227	+ .007	.791	.227	+ .007
.334	.214	− .006	.838	.214	− .006
.395	.216	− .004	.872	.215	− .005
.418	.195	− .025	.921	.233	+ .013
.445	.217	− .003	.948	.217	− .003
.467	.222	+ .002	.971	.222	+ .002
.520	.230	+ .010	.981	.214	− .006
0.559	0.219	−0.001	0.996	0.236	+0.016

error of a single normal $= \pm 0.006$ mag. Any variation must therefore be quite small. Since a_1 has a short period of less than three days, it has seemed worth while to test for the possibility of ellipticity of figure, which would give a sine curve with period one-

half that of the orbital revolution. In Table VII, the combined normals are each the mean of two normals from Table VI, the phase being reduced to the first half of the period. The probable error of a combined normal is ±0.003 mag., so there is no evidence of ellipticity of figure. From the number and size of the residuals in

TABLE VII

COMBINED NORMALS, α_1 *Geminorum*

Phase	Difference of Magnitude	Residual	Phase	Difference of Magnitude	Residual
P	Mag.	Mag.	P	Mag.	Mag.
0.029	0.224	+0.004	0.336	0.214	−0.006
.070	.225	+ .005	.384	.216	− .004
.118	.216	− .004	.420	.214	− .006
.172	.222	+ .002	.446	.217	− .003
.201	.224	+ .004	.469	.222	+ .002
.226	.222	+ .002	0.488	0.225	+0.005
0.262	0.221	+0.001			

TABLE VIII

NORMAL MAGNITUDES, α_2 *Geminorum*

Phase	Difference of Magnitude	Residual	Phase	Difference of Magnitude	Residual
P	Mag.	Mag.	P	Mag.	Mag.
0.005	0.209	−0.011	0.581	0.218	−0.002
.035	.214	− .006	.596	.227	+ .007
.116	.227	+ .007	.659	.223	+ .003
.154	.212	− .008	.706	.221	+ .001
.227	.205	− .015	.753	.216	− .004
.262	.211	− .009	.786	.232	+ .012
.281	.223	+ .003	.814	.234	+ .014
.322	.218	− .002	.828	.224	+ .004
.349	.225	+ .005	.861	.219	− .001
.369	.230	+ .010	.916	.207	− .013
.385	.222	+ .002	.966	.226	+ .006
.473	.230	+ .010	.981	.202	− .018
0.557	0.223	+0.003	0.992	0.228	+0.008

Tables VI and VII, it seems fair to assume that any variability of *Castor* due to α_1 must be less than 0.02 mag., which means that α_1 itself is constant within about 0.06 mag.

The normals on the basis of phase of α_2 are given in Table VIII. Here again the probable error of a single normal comes out ±0.006 mag., and there are certainly no eclipses. There may be a faint

suspicion that the star has a continuous variation, but we may safely put this at less than 0.02 mag., which corresponds to 0.03 mag. in α_2. The normals for both α_1 and α_2 are shown in Fig. 1.

The accordance of the results for *Castor* is disappointing, for the probable error of ±0.006 mag. is no better than in the first work on *Algol*, and not so good as that of *β Aurigae*. The brightness and proximity of *Castor* and *Pollux* made the conditions seem very

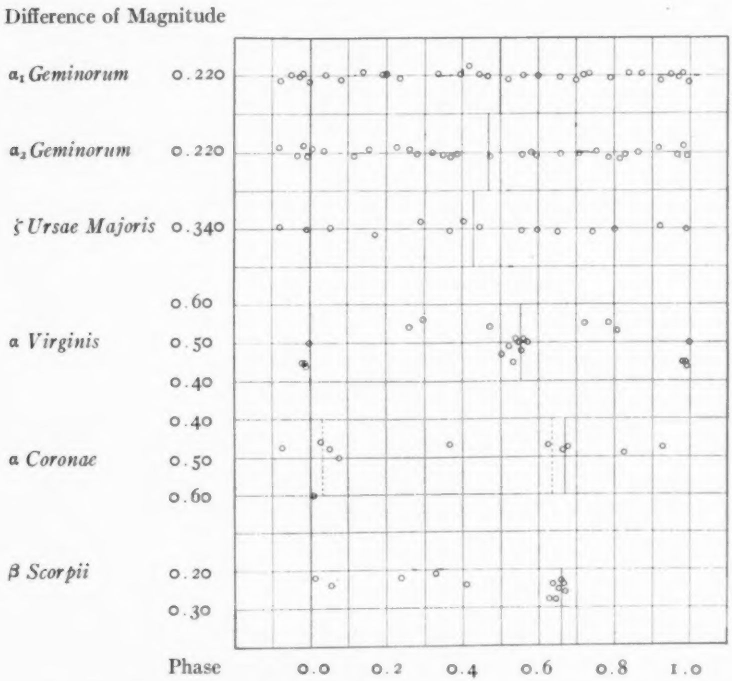


FIG. 1.—Normal magnitudes of spectroscopic binaries. (The vertical scale is uniformly 1 space = 0.10 magnitude.)

favorable, and in spite of the absorption error, I hoped that the probable error of a normal would not exceed ±0.003 mag. Indeed, I am almost forced to the conclusion that the light of *Pollux* is not constant. If the sun, of spectrum Class G, is a variable star with range as much as 5 or 10 per cent, there is every reason to suppose that a Class K star may vary even more. As shown in Table IV, there are nights with systematic discordances of about 0.03 mag.,

and it may easily be that these are due to the variability of *Pollux*. One of the future problems of stellar photometry is a test of the various spectral types, and it will not be surprising to find an increasing tendency to irregular light-variation, as we pass along the scale from spectral classes B to M.

Result for α Geminorum.—An exhaustive test shows that the light of this star is probably constant within 0.02 mag., corresponding to a limit of 0.06 mag. for α_1 *Geminorum*, and to 0.03 mag. for α_2 *Geminorum*.

79 ζ *Ursae Majoris*

H.R. 5054, Mag. 2.40, Spectrum Ap

The orbit is by Vogel[1] and the period by Ludendorff.[2]

P	$20^{d}.536$	e	0.502
T	2415472.84	$\dfrac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	0.486
ω	$101^{\circ}.3$		
	$m_1 = m_2 =$	2.0	

The hypothetical light elements are:

$$\text{Min. I} = \text{J.D. } 2415472.66 + 20^{d}.536 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 8^{d}.79 = 0^{P}.428$$

This was the first spectroscopic binary, discovered at Harvard, and eclipses would be unusually interesting as both components are bright. When the combined light is measured, the presence of the fainter visual component would reduce any variation of the binary by about one-fifth.

The photometric observations were taken in three series. In Series I two comparison stars were used, ε and η *Ursae Majoris*, while in Series II and III the measures all depend upon ε alone. The systematic corrections are:

Series	Correction Mag.
I	+ 0.048
II	0.000
III	+ 0.011

In Table IX, the difference of magnitude is in the sense: ζ *minus* ε. The weights are on the basis of the general accordance of the series.

[1] *Astrophysical Journal*, **13**, 328, 1901.

[2] *Astronomische Nachrichten*, **180**, 276, 1909.

TABLE IX

Observations of ζ *Ursae Majoris*

SERIES I

Date	G. M. T.	Phase	Difference of Magnitude	Weight
1910		P	Mag.	
July 19	$15^h\ 14^m$	0.562	0.358	3
20	17 10	.614	.368	3
23	15 01	.756	.323	3
24	15 36	.806	.333	3
26	15 15	.903	.388	3
29	15 01	.048	.318	3
30	15 00	.097	.338	3
August 2	15 14	.243	.303	3
3	14 48	.291	.298	3
4	14 59	.340	.358	3
7	15 01	.486	.363	3
9	15 08	.584	.303	3
11	14 44	0.681	0.373	3

SERIES II

Date	G. M. T.	Phase	Difference of Magnitude	Weight
1911				
March 19	15 57	0.396	0.351	4
19	17 06	.398	.342	4
19	19 31	.403	.320	4
19	20 49	.406	.319	4
19	21 59	.408	.302	4
20	18 29	.450	.327	4
22	16 41	.544	.344	4
23	16 19	.592	.363	4
24	16 14	.640	.345	4
27	16 37	.787	.322	4
31	16 59	.982	.362	4
31	20 36	.990	.314	4
April 1	16 26	.030	.339	4
9	15 49	.418	.317	4
15	16 15	.711	.353	4
22	16 39	.053	.348	4
23	16 40	.102	.356	4
24	16 36	.150	.371	4
May 2	17 05	.541	.366	4
3	15 08	.586	.330	4
7	16 28	.783	.346	4
10	16 46	0.930	0.324	1

SERIES III

Date	G. M. T.	Phase	Difference of Magnitude	Weight
May 19	16 03	0.367	0.337	5
24	16 04	.610	.339	5
25	15 47	.658	.325	5
27	15 50	.756	.362	5
June 1	15 16	.998	.347	5
6	15 58	.243	.345	5
7	16 04	.292	.330	5
8	16 04	.341	.349	5
10	15 42	.437	.348	3
18	15 51	.827	.361	5
19	15 22	.875	.325	5
21	15 30	0.972	0.309	5

The results are combined into normals in Table X, each normal having a weight of from 12 to 16. The normal magnitudes are represented in Fig. 1. Adopting a constant difference, 0.340 mag., the 13 residuals give a probable error of a single normal $= \pm 0.007$ mag.

Result for ζ Ursae Majoris.—Observations throughout the spectroscopic period give no evidence of eclipses, nor of continuous variation. The star's light is constant within 0.02 or 0.03 mag.

TABLE X

NORMAL MAGNITUDES, *ζ Ursae Majoris*

Phase	Difference of Magnitude	Residual
P	Mag.	Mag.
0.055	0.337	−0.003
.171	.356	+ .016
.292	.323	− .017
.366	.345	+ .005
.404	.321	− .019
.446	.336	− .004
.556	.344	+ .004
.597	.344	+ .004
.649	.349	+ .009
.741	.349	+ .009
.802	.342	+ .002
.920	.333	− .007
0.991	0.341	+0.001

67 α Virginis

H.R. 5056, Mag. 1.21, Spectrum B2

The orbit is by Baker,[1] and both spectra are measurable.

P	4^d01416	$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	0.823
T	2417955.846		
ω	328°	$m_1 \sin^3 i$	9.6
e	0.10	$m_2 \sin^3 i$	5.8

The hypothetical light-elements are:

$$\text{Min. I} = \text{J.D. } 2417957.094 + 4^d01416 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 2^d224 = 0^P554$$

With the selenium photometer, two comparison stars have been used: *4 γ Corvi, H.R.* 4662, mag. 2.78, spectrum B8; *27 β Librae,*

[1] *Publications of the Allegheny Observatory*, 1, 72, 1909.

TABLE XI

OBSERVATIONS OF *α Virginis*

Date	G.M.T.	Phase	Difference of Magnitude	Comparison Star
1912		P	Mag.	
March 25	17^h 28^m	0.309	0.54	γ
25	17 52	.313	.56	γ
April 3	15 32	.531	.42	γ
3	16 00	.536	.48	γ
3	16 26	.540	.50	γ
3	17 02	.546	.48	γ
3	17 26	.550	.45	γ
3	18 00	.556	.47	γ
7	15 45	.529	.46	γ
7	16 10	.534	.46	γ
7	16 34	.538	.51	γ
7	16 58	.542	.52	γ
7	17 21	.546	.52	γ
7	17 44	.550	.51	γ
7	18 16	.556	.53	β
7	18 38	.559	.53	β
7	19 02	.564	.55	β
7	19 36	.570	.49	β
7	19 58	.573	.52	β
8	16 12	.783	.53	γ
8	16 35	.787	.57	γ
8	16 58	.791	.52	γ
8	18 51	.811	.54	β
8	19 22	.816	.54	β
15	15 25	.519	.47	γ
15	19 13	.558	.44	β
22	15 17	.261	.54	γ
22	15 40	.265	.54	γ
22	16 03	.269	.58	γ
May 7	14 20	.988	.34	γ
7	14 43	.992	.41	γ
7	15 06	.996	.48	γ
9	16 39	.510	.47	β
9	17 03	.514	.45	β
9	17 22	.518	.55	β
19	15 27	.989	.39	β
19	16 24	.999	.48	β
19	16 48	.003	.47	β
23	14 39	.977	.46	γ
23	15 18	.984	.50	β
23	15 42	.988	.51	β
23	16 18	.994	.57	β
31	16 32	.990	.52	β
June 2	14 44	.469	.56	β
2	15 07	.473	.48	β
2	15 30	.477	.57	β
2	15 54	.481	.48	β
4	15 40	.977	.43	β
4	16 04	.981	.46	β
7	15 36	.724	.54	β
7	16 06	.729	.52	β
11	14 59	0.714	0.60	β

H.R. 5675, mag. 2.74, spectrum B8. It had been my intention to cut down the light of *Spica* by means of a screen, but a few experiments showed that it was feasible to shorten the exposures. The bright star was therefore given a 3-second exposure, as against 10 seconds for the comparison stars. This procedure is not without some risk, but the outcome seems to be satisfactory. On a good night, as the stars approached the meridian, the measures could be made with γ *Corvi* for several hours. Then when the absorption correction became too large, a pause of an hour or so was necessary until β *Librae* rose to a sufficient altitude.

In Table XI the difference of magnitude is in the sense: comparison star 10 seconds *minus Spica* 3 seconds. Each observation comprises two sets of measures. A discussion of the suitable observations gave not more than 0.01 mag. difference between the two comparison stars, and they are assumed to be equal. These measures, while not extending over the entire period, are sufficiently numerous to combine into normals, as shown in Table XII.

TABLE XII

NORMAL MAGNITUDES, α *Virginis*

Phase	Difference of Magnitude	Phase	Difference of Magnitude
P	Mag.	P	Mag.
0.263	0.54	0.560	0.51
.297	.56	.571	.50
.473	.54	.722	.55
.502	.47	.785	.55
.522	.49	.806	.53
.533	.45	.979	.45
.540	.51	.987	.45
.547	.50	.990	.44
0.554	0.48	0.998	0.50

The normals are represented in Fig. 1, and it will be seen that either *Spica* is a variable, or some extraordinary error has crept in. The star was measured as faint near both predicted minima, and the drop was greater near primary minimum. The same variation is shown with each comparison star. A curious feature is that the eclipses are coming ahead of the times predicted from the spectroscopic elements, but a similar discrepancy in the same direction has been noted in other stars. A discussion of the probable errors

shows that the deviations from constant light are much greater than could be expected from the internal agreement of the measures. However, this is not a question to be settled by probable errors. The measures are admittedly difficult because of the low altitude of the stars, but it is my deliberate judgment that *Spica* is a variable.

Needless to say, it will be worth while to continue the observations, and fix the variation beyond a possibility of doubt. In *Spica* we have again the favorable case of an eclipsing variable with spectroscopic elements for both components, and in view of the early B2 spectrum, the computed dimensions in the system will be of special interest and importance.

Result for α Virginis.—The measures indicate that this is an eclipsing variable with two minima, the extreme range being something like 0.10 magnitude.

5 α Coronae Borealis

H.R. 5793, Mag. 2.31, Spectrum A0

There are two independent orbits by Cannon[1] and Jordan.[2]

	Cannon	Jordan
P	$17^d.355$	$17^d.36$
T	2417725.054	2417742.55
ω	$303^\circ.68$	$312^\circ.2$
e	0.277	0.387
$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	0.057	0.060

The hypothetical light-elements from Cannon are:

$$\text{Min. I} = \text{J.D. } 2417731.09 + 17^d.355 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 10^d.41 = 0^P.600$$

and from Jordan:

$$\text{Min. I} = \text{J.D. } 2417730.04 + 17^d.36 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 11^d.64 = 0^P.670$$

There is a difference of more than a whole day in the time of primary minimum predicted from the two orbits; but, with the different

[1] *Journal of the Royal Astronomical Society of Canada*, **3**, 152, 1909.

[2] *Publications of the Allegheny Observatory*, **1**, 89, 1909.

periods, this is reduced to about half a day at the epoch of the photometric observations.

The possibility of testing this star was overlooked for some time perhaps because the period of 17 days is not especially short. A few measures were taken in 1911, and as some were pretty close to minimum phase, it was thought that there was little need of further tests. However, on April 7, 1912, we had a fine night, and were making the most of it in observing several stars. Knowing that *α Coronae* was very near a predicted minimum, we started to measure it at about 14^h local mean time. I was at once amazed to find it faint, and the observations were therefore continued until stopped by dawn. On the next night, the star was back at normal light, and a second opportunity to test the minimum has not occurred, though measures have been made at other points along the light-curve.

The measures in Table XIII are all referred to *η Ursae Majoris*, though some of the earlier comparisons were with *ε Boötis*. The difference of magnitude is in the sense: *α Coronae* minus *η Ursae Majoris*. Each observation comprises three sets of measures, and there is practically no difference between the means for the two years.

The normals are shown in Fig. 1, where the dotted lines represent Cannon's minima. Excluding the first normal, we get 0.477 mag. for the mean. The residuals give a probable error of a single normal $= \pm 0.008$ mag., and the first residual, due to the eclipse, is fifteen times the probable error. If this minimum of *α Coronae* were a phenomenon observed once for all, it might be worth while to give in detail all of the measures on the one critical night. But we have the spectroscopic period, and anybody may test for the light change at his convenience.

There is no evidence of a secondary minimum, though there are three normals close to this phase. As only one spectrum is visible, the companion may not be very bright. It is not probable, therefore, that we shall be able to derive the approximate dimensions in this system, though they would be interesting since *α Coronae* is a member of the extended *Ursa Major* group.

TABLE XIII

OBSERVATIONS OF α *Coronae Borealis*

Date	G.M.T.	Phase		Difference of Magnitude	Remarks
		Cannon	Jordan		
1911		P	P	Mag.	
May 24........	$17^h\ 05^m$	0.585	0.622	0.48	ε Boötis
31........	15 54	.986	.022	.44	ε Boötis
31........	18 15	.991	.028	.48	ε Boötis
31........	19 08	.993	.030	.46	
31........	20 48	.998	.034	.46	
June 1........	16 06	.044	.080	.50	
1........	16 48	.046	.082	.52	ε Boötis
13........	15 42	.734	.770	.49	
18........	16 34	.024	.060	.49	
1912					
April 7........	20 42	.975	.006	.59	Eclipse
7........	21 23	.976	.007	.61	Eclipse
7........	22 07	.978	.009	.60	Eclipse
8........	20 08	.031	.062	.48	
8........	20 50	.033	.064	.48	
18........	20 59	.609	.640	.45	
18........	21 42	.611	.642	.48	
22........	16 43	.830	.861	.51	
23........	19 58	.895	.926	.49	
23........	20 40	.897	.927	.46	
May 2........	16 40	.406	.436	.46	
2........	17 18	.407	.438	.44	
5........	16 21	.578	.608	.48	
6........	18 26	.640	.671	.48	
6........	19 02	.642	.672	.49	
6........	19 42	.643	.674	.50	
6........	20 18	.645	.676	.45	
9........	18 11	.813	.843	.47	
June 2........	16 42	0.192	0.222	0.50	

TABLE XIV

NORMAL MAGNITUDES, α *Coronae Borealis*

Phase		Difference of Magnitude	Residual
Cannon	Jordan		
P	P	Mag.	Mag.
0.976	0.008	0.600	+0.123
.990	.027	.460	− .017
.018	.052	.477	.000
.041	.075	.500	+ .023
.335	.365	.467	− .010
.591	.623	.470	− .007
.631	.662	.483	+ .006
.644	.675	.475	− .002
.792	.824	.490	+ .013
0.896	0.926	0.475	−0.002

Result for α Coronae.—Observations were made at principal minimum on one night under first-class conditions. The star was then about 0.12 mag. fainter than normal, and is therefore an eclipsing variable.

8 β Scorpii

H.R. 5984, Mag. 2.90, Spectrum B1

The orbit is by Daniel and Schlesinger.[1] A second orbit by Duncan[2] is practically identical, and gives the same times of eclipses. Two spectra are visible.

P	$6^d.8283$	$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	1.26
T	2419163.923		
ω	$20^\circ.09$	$m_1 \sin^3 i$	13.0
e	0.270	$m_2 \sin^3 i$	8.3

The hypothetical light-elements are:

$$\text{Min. I} = \text{J.D. } 2419164.74 + 6^d.8283 \cdot E$$
$$\text{Min. II} - \text{Min. I} = 4^d.510 = 0^P.660$$

Although this star is rather far south it can be compared conveniently with *δ Scorpii*. The difference of magnitude is in the sense: *β minus δ*. Each observation comprises two sets.

These normals are shown in Fig. 1. The normal at phase $0^P.012$ does not indicate an eclipse, but the results near secondary minimum make the star faint. Here, as in the case of *Spica*, we should have to assume that the eclipses are coming ahead of time.

Result for β Scorpii.—The results are not conclusive, but lead to a suspicion that the star is an eclipsing variable with small range.

DISCUSSION

With the stars tested with the selenium photometer, we may include *β Aurigae*[3] and *δ Orionis*. I have finished the light-curve for the latter, but some further spectroscopic data are necessary for a final discussion. Let it suffice to state that *δ Orionis* is an eclipsing variable with two minima, the total range being about 0.12

[1] *Publications of the Allegheny Observatory*, **2**, 135, 1912.

[2] *Lowell Observatory Bulletin*, **2**, 21, 1912.

[3] *Astrophysical Journal*, **34**, 112, 1911.

TABLE XV

OBSERVATIONS OF β *Scorpii*

Date	G.M.T.	Phase	Difference of Magnitude
1912		P	Mag.
March 25	$21^h\ 31^m$	0.326	0.20
25	21 48	.328	.22
25	22 05	.330	.20
30	20 45	.054	.22
30	21 04	.056	.26
April 3	20 48	.640	.19
3	21 10	.642	.26
3	21 36	.645	.28
3	21 56	.647	.30
3	22 22	.649	.30
10	18 58	.654	.25
10	19 16	.656	.21
10	19 34	.657	.23
10	19 52	.659	.24
10	20 14	.661	.23
10	20 34	.663	.30
10	20 56	.666	.22
10	21 15	.668	.19
10	21 34	.670	.28
10	21 53	.671	.23
26	21 06	.010	.21
26	21 28	.012	.20
26	21 43	.014	.24
May 14	17 40	.625	.30
14	18 01	.627	.26
14	18 22	.629	.27
14	18 42	.631	.26
14	19 04	.634	.28
25	17 41	.236	.24
25	18 02	.238	.21
25	18 21	.240	.20
June 2	17 40	.408	.21
2	18 00	0.410	0.26

TABLE XVI

NORMAL MAGNITUDES, β *Scorpii*

Phase	Difference of Magnitude	Phase	Difference of Magnitude
P	Mag.	P	Mag.
0.012	0.22	0.635	0.24
.055	.24	.645	.28
.238	.22	.653	.25
.328	.21	.659	.23
.409	.24	.666	.24
0.627	0.28	0.671	0.26

magnitude. We have then eleven stars which may be divided as shown in Table XVII. It seems fair to place *β Scorpii* in the first list, and *α Virginis* in the second. It appears then that 4 or 5 out of the 11 stars are eclipsing variables. Among the constant stars, the only one with a short period and large mass function is *β Scorpii*. Of the variables, all are favorable cases except perhaps *α Coronae*.

TABLE XVII

Constant Stars				Eclipsing Stars			
Star	Period	Spectrum	$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	Star	Period	Spectrum	$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$
	d				d		
α Andromedae	96.67	Ao	0.18	*β Aurigae*	3.96	Ap	0.54
α Aurigae	104.02	Go	0.18	*δ Orionis*	5.73	Bo	0.60
ι Orionis	29.14	Oe5	1.14	*α Virginis*	4.01	B2	0.82
$α_1$ Geminorum	2.93	Ao	0.0097	*α Coronae*	17.36	Ao	0.06
$α_2$ Geminorum	9.22	Ao	0.0015				
ζ Ursae Majoris	20.54	Ap	0.49				
β Scorpii	6.83	B1	1.26				

Of course, we can draw no general conclusions from a few stars, but the desirability of testing other objects is apparent. This has already been done with success by Hertzsprung[1] and Shapley.[2] I have shown[3] that any reasonable assumption as to the densities and size of companions in short-period binaries leads to a large proportion of systems whose eclipses may be detected. In fact, one is somewhat appalled at the number of variables which are awaiting discovery. The known spectroscopic binaries are increasing by leaps and bounds; and in addition to the eclipsing variables among these, there are no doubt many of other periodic types, not to mention those with irregular light-changes similar to the sun.

The tests for small light-variations can be made only with the most accurate forms of photometer, and while the possibilities of the selenium method are still very great, it looks as if the

[1] *Astronomische Nachrichten*, **195**, 307, 1913.

[2] *Ibid.*, **196**, 383, 1913.

[3] *Astrophysical Journal*, **34**, 105, 1911.

photo-electric cell would furnish the means for the next improvements. About a year ago, Professor W. F. Schulz[1] tried a potassium cell on our 12-inch telescope with encouraging results, and the experiments are being continued by Messrs. Kunz, Schulz, and myself. As is well known, photo-electric cells like those of Elster and Geitel have been used successfully by Meyer and Rosenberg[2] at Tübingen, and by Guthnick[3] at Berlin-Babelsberg.

The observations described in the present paper were secured with the assistance of Messrs. Percy F. Whisler and H. F. Zoller. I am also indebted to Mr. J. D. Bond for checking some of the reductions. This work is a portion of that made possible by several grants from the Rumford Fund of the American Academy of Arts and Sciences.

University of Illinois Observatory
Urbana, Ill.
February 6, 1914

[1] *Astrophysical Journal*, **38**, 187, 1913.

[2] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **48**, 210, 1913.

[3] *Astronomische Nachrichten*, **196**, 357, 1913.

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AUTHORS

Recent Publications

OF

The University of Chicago Press

Chicago and the Old Northwest, 1673–1835. By Milo Milton Quaife, Professor of History in the Lewis Institute of Technology.

488 pages, 8vo, cloth; $4.00, postpaid $4.23

This book recounts, in a manner at once scholarly and dramatic, the early history of Chicago. Important as this subject is, it is not treated solely for its own sake. The author's larger purpose has been to trace the evolution of the frontier from savagery to civilization. From the point of view of Chicago and the Northwest alone the work is local in character, although the locality concerned embraces five great states of the Union; in the larger sense its interest is as broad as America, for every foot of America has been at some time on the frontier of civilization.

This task has never before been performed in an adequate way. The one really brilliant historian of Illinois, Mr. Edward G. Mason, died with only a few fragments of his great work completed, and no one has yet come forward to take his place. It is believed that this book will take rank as the standard history of Chicago in the early days.

Chicago Tribune. A history of the beginnings of Chicago which, because it is scientific and based upon records rather than upon tradition, may overturn existing chronologies on the same subject.

Animal Communities in Temperate America. A Study in Animal Ecology. By Victor Ernest Shelford, Instructor in Zoölogy in the University of Chicago.

380 pages, 8vo, cloth; $3.00, postpaid $3.22

This volume by Dr. Shelford presents the principles of field ecology, illustrated by the more widely distributed animal habitats of the eastern half of temperate North America, and the aquatic habitats of a much larger territory. Six chapters deal with general principles.

In several chapters animal communities of lakes, streams, swamps, forests, prairies, and various soils and topographic situations are considered from the point of view of modern dynamic ecology. A very valuable feature of the book is the

three hundred figures of widely distributed animals chosen to represent the chief types of animal communities and their characteristic modes of life.

Artificial Parthenogenesis and Fertilization. By Jacques Loeb, Member of the Rockefeller Institute for Medical Research.

318 pages, 12mo, cloth; $2.50, postpaid $2.68

This new work presents the first complete treatment of the subject of artificial parthenogenesis in English. Professor Loeb published four years ago a book in German under the title *Die chemische Entwicklungserregung des tierischen Eies.* Mr. W. O. R. King, of the University of Leeds, England, translated the book into English, and the translation has been revised, enlarged, and brought up to date by Professor Loeb. It gives, as the author says in the preface, an account of the various methods by which unfertilized eggs can be caused to develop by physico-chemical means, and the conclusions which can be drawn from them concerning the mechanism by which the spermatozoon induces development. Since the problem of fertilization is intimately connected with so many different problems of physiology and pathology, the bearing of the facts recorded and discussed in the book goes beyond the special problem indicated by the title.

The Mechanistic Conception of Life. Biological Essays by Jacques Loeb, Member of the Rockefeller Institute for Medical Research.

238 pages, 12mo, cloth; $1.50, postpaid $1.65

The achievements of Professor Jacques Loeb in the field of experimental biology insure any book of his a wide reading. His experimental work at the universities of Chicago and California, as well as in his present position, gives this volume an especial significance. Professor Loeb here presents many of the current problems in biology, and discusses the question whether the phenomena of life can be unequivocally explained in physico-chemical terms.

Johns Hopkins Hospital Bulletin. All eager to know more of the origin of life will find no modern book of its size nearly so instructive or inspiring.

A Manual for Writers. By John M. Manly, Head of the Department of English in the University of Chicago, and John A. Powell, of the University of Chicago Press.

226 pages, 12mo, cloth; $1.25, postpaid $1.35

A book designed to aid authors and all others who are concerned with the writing of English. It aims to answer the practical questions that constantly arise in the preparation of

manuscripts for the printer, business letters, and any sort of composition where correctness of form is an important element.

It treats in a clear and convenient way the matters of grammar, spelling, and general form which writers need most to be informed about, and gives full directions on the preparation of "copy" for the printer and the correcting of proof. The chapter on letter-writing is unique and of especial value in its practical suggestions.

C. E. Raymond, Vice-President of the J. Walter Thompson Co. It seems to me to be the most comprehensive and comprehensible of any of the works on this subject which I have had the pleasure of seeing.

The Elements of Debating: A Manual for Use in High Schools and Academies. By Leverett S. Lyon, of the Joliet High School.

148 pages, 12mo, cloth; $1.00, postpaid $1.07

The aim of the book is to put the well-established principles of the art of debating within the reach of young students. It therefore presents the elements of public speaking so freed from technicality that the student may assimilate them in the shortest possible time and with the least possible interpretation by the teacher.

The book consists of ten chapters and a number of appendices. Each chapter is preceded by an analysis of the subject and followed by a series of suggested exercises. The whole subject is treated in a direct, practical way with the greatest possible clearness, and with illustrations drawn from subjects familiar and interesting to high-school boys. It is entirely modern in that it lays stress on efficiency, rather than on theoretical perfection. Illustrations are given from some of the most effective arguments ever written, and a list of suggested topics is added in an appendix.

London in English Literature. By Percy Holmes Boynton, Assistant Professor of English Literature in the University of Chicago.

358 pages, 8vo, cloth; $2.00, postpaid $2.17

This volume differs from all other volumes on London in that it gives a consecutive illustrated account of London not from the point of view of the antiquarian but from that of the inquiring student of English literary history.

It deals with ten consecutive periods, characterized in turn by the work and spirit of Chaucer, Shakespeare, Milton, Dryden, Addison, Johnson, Lamb, Dickens, and by the qualities of Vic-

torian and contemporary London. The emphasis is thus distributed over history and given largely to the richer and more recent literary periods. The temper of each epoch is discussed, and then in particular those literary works which are intimately related to certain localities in London.

The work contains four maps and forty-three other illustrations, selected from the best of a great fund of material. As further aids to the student or the general reader, the sources of all material are indicated by footnotes and lists of illustrative reading are appended to each chapter. There are also an appendix with detailed references to illustrative novels, and a carefully compiled index.

The Springfield Republican. It would seem as if Mr. Boynton's book would make strong appeal to everybody who has been in London, and a good many more who would like to go there; . . . . he has caught its spirit and presented it here.

Social Programmes in the West. (The Barrows Lectures.) By Charles Richmond Henderson, Head of the Department of Practical Sociology in the University of Chicago.

212 pages, 8vo, cloth; $1.25, postpaid $1.38

The Barrows Lectures (1912–13) delivered with so much success in the Far East by Professor Henderson are included in this volume, which is also published in India by the Macmillan Company. The subjects of the lectures are as follows: "Foundations of Social Programmes in Economic Facts and in Social Ideals," "Public and Private Relief of Dependents and Abnormals," "Policy of the Western World in Relation to the Anti-Social," "Public Health, Education, and Morality," "Movements to Improve the Economic and Cultural Situation of Wage-Earners," and "Providing for Progress." The author, in his preface, says that "the necessity of selecting elements from the social activities of Europe and America, which might have value in the Orient under widely different conditions, compelled a consideration of the materials from a new point of view." The introduction includes, besides a syllabus of the six lectures, the Letter of Commission from the officers of the three great international associations for labor legislation, asking the lecturer to present their aims wherever it was possible in India, China, and Japan. There is also included a statement by Professor E. Fuster, of Paris, of the aims of the international associations on social legislation.

Boston Transcript. The lectures will be of value to American readers in that they concisely place before them the social problem in its most fundamental aspects.

Christian Faith for Men of Today. *By E. Albert Cook, Professor of Systematic Theology in the Congregational College of Canada.*

276 pages, 12mo, cloth; $1.25, postpaid $1.35

This volume contains a clear and reasonable interpretation of life from the Christian point of view, and some definite formulation of those beliefs that have proved most effectual in the development of individual character and in the promotion of the welfare of society. Professor Cook believes that Christianity is the religion which best meets the needs of all races and classes. The convenient arrangement of the material in numbered paragraphs, the valuable appendices, and the general systematic treatment of the subject commend the book as a popular text for college men and women, and for adult classes and clubs as well as for general reading.

Watchman. It is written for those who feel the currents of thought of this age and have conceptions framed by science, sociology, and history, and need something more acceptable to their reason than current traditional conceptions. The treatment of the Scriptures is reverent but free, and does not disturb faith, but resets it according to modern conceptions.

Materials for the Study of Elementary Economics. *By Leon Carroll Marshall, Chester Whitney Wright, and James Alfred Field, of the Department of Political Economy in the University of Chicago.*

946 pages, 8vo, cloth; $2.75, postpaid $3.00
Special Library Edition, postpaid $4.80

This volume of nearly a thousand pages is intended to supply to students of elementary economics a collection of readings, illustrating the working of economic principles in actual life. The material is drawn from the most various sources—books, magazines, newspapers, commission reports, court decisions, corporation charters, government circulars, etc. The selections have been carefully edited, so as to eliminate unsuitable material and present the documents in the most usable form.

The volume, which is intended to be used in connection with any standard text, has already been adopted by many of the leading institutions of the country.

Professor John Bauer, Cornell University. It ought to have a large demand from all classes of institutions.

Professor Charles C. Arbuthnot, Western Reserve University. *Materials for the Study of Elementary Economics* is altogether the most promising collection of illustrative material I have ever seen.

Outlines of Economics, Developed in a Series of Problems. By Members of the Department of Political Economy of the University of Chicago.

160 pages, interleaved, 12mo, cloth; $1.00, postpaid $1.13

This book is an attempt on the part of its authors to make some advances in the direction of improving the current methods of teaching the elementary course in economics. The ideals which have shaped the character of the book are: (1) A belief that the elementary course in economics offers exceptional opportunities for training in thinking and reasoning and that the inductive-problem method here used is the one best adapted to accomplish this end; (2) a desire to connect the theoretical principles of economics with the actual facts and with problems of the business world, and to induce the student to apply his knowledge of that world to the subject of study.

The result is a careful, analytical syllabus of the subjects usually covered in the introductory course, accompanied by some 1,200 questions and problems, designed: (*a*) to afford set problems for written work; (*b*) to guide the student in his reading, while fostering independent thinking; (*c*) to give direction to classroom discussion.

Nation. In their *Outlines of Economics, Developed in a Series of Problems,* three members of the Department of Political Economy in the University of Chicago have performed with remarkable thoroughness and grasp a task of great difficulty.

My Life. By August Bebel. (*With a Portrait.*) *An Autobiography of the Famous Leader of the Social Democratic Party in Germany.*

344 pages, 8vo, cloth; $2.00, postpaid $2.14

This autobiography contains the story, from the inside, of the rise of the German trades-unions, and throws many interesting sidelights on the politics of Bismarck and Lassalle. Up to the time of his death in August, 1913, Bebel was the molding influence of the Social Democratic party, which is, even under the restricted franchise, a power in the German state. Bebel gives a vivid description of the poverty of his early life and of his wanderings as a craftsman in search of work all through Germany—the Germany before the wars of 1864, 1866, and 1870–71, and before the Unification. Soon after his election to the Reichstag he was convicted of high treason for the expression of his views on the Franco-German War and had to spend some years in prison. The international reputation of the author, the extreme frankness with which he writes, and the striking suc-

cesses of the movement with which he is identified make this a human document of remarkable interest and significance.

The Nation. He is a rare narrator. . . . . He humanizes a great cause, and by so doing makes it intelligible, and perhaps even appealing, to thousands who would otherwise pass by on the other side.

Francesco Petrarca and the Revolution of Cola di Rienzo. A Study in the History of Rome during the Middle Ages. By Mario Emilio Cosenza, Instructor in Latin in the College of the City of New York.

335 pages, 12mo, cloth; $1.50, postpaid $1.60

In these pages the author draws a picture of Petrarch as a statesman, believing that even if Petrarch had never written a sonnet in praise of Laura he would still be dear to many generations of Italians for having been the first real Italian patriot—a man who was not bounded by narrow partisanship but who through a long and active life was wholly devoted to the cause of a unified Italy. Dr. Cosenza has chosen for special treatment Petrarch's relations with Cola di Rienzo, because they constitute a story that is virtually a chapter in the history of Rome during the Middle Ages. The material of the present volume is drawn chiefly from Petrarch's letters, from the extremely important correspondence of Cola di Rienzo, and from the equally important archives of the Roman church. Nearly all this material is new to the English language. The notes are detailed enough to make clear Petrarch's many allusions. The book is written with the charm of a vital scholarship and with intimate feeling for its subject, and the incidents connected with the lives of the two great Italians who lived centuries in advance of their times have a remarkable variety and interest.

Boston Evening Transcript. As we read these extracts from the letters of Petrarca, and the scholarly notes that accompany them, we are convinced that, whatever the opinion of the papal court, Petrarca was a great statesman.

American Poems. Selected and Edited with Illustrative and Explanatory Notes and a Bibliography. By Walter C. Bronson, Litt.D., Professor of English Literature in Brown University.

680 pages, 12mo, cloth; $1.50, postpaid $1.68

The book offers a most carefully chosen and well-balanced presentation of the poetic works of Americans, covering the entire period of our history. For the teacher as well as the student the value of the work is greatly enhanced by the comprehensive Notes, Bibliography, and Indices. It is believed that

the book will have the wide popularity of Professor Bronson's earlier collection, *English Poems*, which has been adopted by all leading American colleges.

The Dial. The resources of the special collections of Brown University have supplied the editor with the best authorities for accurate texts, and have made possible the widest range of selections.

Education. Professor Bronson has done a real service to teachers and students of literature. . . . . It is a truly adequate presentation of American poetry.

The Courts, the Constitution, and Parties. Studies in Constitutional History and Politics. By Andrew C. McLaughlin, Professor of History in the University of Chicago.

308 pages, 12mo, cloth; $1.50, postpaid $1.63

A volume of peculiar interest at this time, when the courts and political parties are subject to general criticism. The discussion is especially significant as coming from a lifelong student of constitutional questions, whose work at the University of Michigan, the Carnegie Institution of Washington, and the University of Chicago is so widely known. The point of view is historical. Though the articles are scientific they are intended for the reader who is interested in public affairs rather than for the professional student. The work consists of five papers, the first of which discusses the power of a court to declare a law unconstitutional. Two of the papers deal with the growth and essential character of political parties, and are followed by one on the history of differing theories of the federal Union. The work concludes with a discussion of the written constitution in some of its historical aspects, taking up the origin of these documents and the problems of their maintenance and interpretation in the development of the modern popular state.

Harvard Law Review. To the reviewer the two papers first mentioned seem to be contributions of great and permanent value to the discussion of their topic. . . . . The style of these essays is easy and delightful and their argument sane, thoughtful, and persuasive.

Political Science Quarterly. Professor McLaughlin, in this most important essay under review, has surveyed the field anew, and with rare appreciation of the purport and the weight of evidence has contributed a judgment which may well be regarded as definitive.

Heredity and Eugenics. By John M. Coulter, William E. Castle, Edward M. East, William L. Tower, and Charles B. Davenport.

312 pages, 8vo, cloth; $2.50, postpaid $2.70

Leading investigators, representing the University of Chicago, Harvard University, and the Carnegie Institution of Washington,

have contributed to this work. Great care has been taken by each contributor to make clear to the general reader the present position of evolution; the results of experiments in heredity in connection with both plants and animals; and the enormous value of the practical application of these laws in breeding and in human eugenics. The volume is profusely illustrated.

British Medical Journal. Those who are desirous of arriving at an estimate of the present state of knowledge in all that concerns the science of genetics, the nature of the experimental work now being done in its various departments, ... and the prospects, immediate or remote, of important practical applications, cannot do better than study *Heredity and Eugenics.*

The Assyrian and Babylonian Letters Belonging to the Kouyunjik Collections of the British Museum, Parts XII and XIII. Edited by Robert Francis Harper, Professor of the Semitic Languages and Literatures in the University of Chicago.

Part XII, 134 pp., 4 colotypes. 8vo, cloth; \$6.00, postpaid \$6.14
Part XIII, 135 pp., 4 colotypes. 8vo, cloth; \$6.00, postpaid \$6.14

These two parts present further instalments of the texts of the Letter Tablets now to be found in the Kouyunjik Collections of the British Museum. Part XII contains the texts of one hundred tablets; Part XIII the texts of ninety-seven tablets. Part I of this series was published in 1892, and the series is now approaching completion. Part XIV is in preparation, and it is hoped to bring the remaining texts into Parts XIV, XV, and XVI. In Part XVII it is planned to give the results of a final collation of all of the texts, and to suggest many restorations which have not been included in the texts themselves. After the completion of the textual work, the author plans to continue with transliterations, translations, and philological notes. When completed this series will be a most valuable sourcebook for the history of the Sargon period.

PUBLICATIONS OF THE CAMBRIDGE UNIVERSITY PRESS

THE **University of Chicago Press** has become the American agent for the scientific journals and the following books issued by the Cambridge University Press of England:

BOOKS

The Life and Correspondence of Philip Yorke, Earl of Hardwicke, Lord High Chancellor of Great Britain. By Philip C. Yorke, M. A. Oxon., Licencié-ès-Lettres of the University of Paris.

Royal 8vo. Three Vols., with six illustrations. Vol. I, pp. 702; Vol. II, pp. 606; Vol. III, pp. 662. Price $13.50, postpaid $14.22

This solid and significant work is based on the Hardwicke and Newcastle manuscripts and, in addition to the life of Lord Hardwicke, gives the whole history of the Georgian period from 1720 to 1764. An account of the great judge's work in the King's Bench and in Chancery is included. The characters and careers of Walpole, Newcastle, Henry Pelham, the elder Pitt, Henry Fox, the Duke of Cumberland, George II, and George III and various incidents—such as the fall of Walpole, the Byng catastrophe, and the struggle between George III and the Whigs—appear in a clearer light, which the author, by aid of original papers and manuscripts, has been enabled to throw upon them. These documents are now published, or brought together and annotated, for the first time.

The North American Review. It corrects errors of previous ill-informed or prejudiced biographers of Lord Hardwicke, and presents an apparently just portrait of a really eminent man, together with a wealth of historical information.

The Genus Iris. By William Rickatson Dykes. With Forty-eight Colored Plates and Thirty Line Drawings in the Text.

254 pages, demi folio, half morocco; $37.50, postpaid $38.36

This elaborate and artistic volume brings together the available information on all the known species of Iris. The account of each includes references to it in botanical literature and a full description of the plant, together with observations on its peculiarities, its position in the genus, its value as a garden plant, and its cultivation. As far as possible the account of the distribution of each species is based on the results of research in the herbaria of Kew, the British Museum, the Botanic Gardens

of Oxford, Cambridge, Berlin, Paris, Vienna, and St. Petersburg, and the United States National Museum at Washington.

The most striking feature of the book is the forty-eight life-size colored plates, reproduced from originals drawn from living plants—making it a volume of great beauty as well as of scientific importance.

The American Florist. Lovers of irises owe a huge debt of gratitude to William Rickatson Dykes, who after years of labor has produced a magnificent work on these plants. . . . . Mr. Dykes combines the scientist's analytical skill with all the grower's enthusiasm.

The Florists' Review. If anything else could be added to the book that would really increase its beauty or its scientific value or its practical utility, the present reviewer is curious to know what that addition could be.

Byzantine and Romanesque Architecture. By Thomas Graham Jackson, R.A. Two Volumes, with 165 Plates and 148 Illustrations.

Vols. I and II, each 294 pages, crown quarto, half vellum; two vols. $12.50, postpaid, $13.25

This work contains an account of the development in Eastern and Western Europe of Post-Roman architecture from the fourth to the twelfth century. It attempts not merely to describe the architecture, but to explain it by the social and political history of the time. The description of the churches of Constantinople and Salonica, which have a special interest at this time, is followed by an account of Italo-Byzantine work at Ravenna and in the Exarchate, and of the Romanesque styles of Germany, France, and England. Most of the illustrations are from drawings by either the author or his son, and add great artistic value to the volumes.

The Nation. The two volumes must surely take their place among the standard classics of every architectural library.

The Duab of Turkestan. A Physiographic Sketch and Account of Some Travels. By W. Rickmer Rickmers. With 207 Maps, Diagrams, and Other Illustrations.

580 pages, royal 8vo, cloth; $9.00, postpaid $9.44

A record of exploration of a little-known region, combined with some elementary physiography. The book discusses the various geographical elements in the natural organic system of the Duab of Turkestan (or Land between the Two Rivers) between the Oxus and the Jaxartes, the information being strung on the thread of a highly interesting story of travel and mountain exploration. The author was at great pains to obtain typical views of physical features such as mountains, valleys, and glaciers,

and also of vegetation, village life, and architecture; and there are many diagrams for a clearer understanding of the text.

The book is especially suitable for colleges, libraries, and schools, and for all students or teachers of physical geography and natural science.

The Journal of Geography. The author's delicate touches of humor, his picturesque language in description, and his knowledge of physiography and climatology, ... all contribute materially to the excellence of the book. Much attention is given to physiographic processes and features, but the splendid half-tones tell the story better than words.

JOURNALS

Biometrika. A journal for the statistical study of biological problems. Edited by KARL PEARSON. Subscription price, $7.50 a volume; single copies, $2.50.

Parasitology. Edited by G. H. F. NUTTALL and A. E. SHIPLEY. Subscription price, $7.50 a volume; single copies, $2.50.

Journal of Genetics. Edited by W. BATESON and R. C. PUNNETT. Subscription price, $7.50 a volume; single copies, $2.50.

The Journal of Hygiene. Edited by G. H. F. NUTTALL. Subscription price, $5.25 a volume; single copies, $1.75.

The Modern Language Review. Edited by J. G. ROBERTSON, G. C. MACAULAY, and H. OELSNER. Subscription price, $3.00 a volume; single copies, $1.00.

The British Journal of Psychology. Edited by W. H. R. RIVERS and C. S. MYERS. Subscription price, $3.75 a volume.

The Journal of Agricultural Science. Edited by PROFESSOR R. H. BIFFEN, A. D. HALL, and PROFESSOR T. B. WOOD. Subscription price, $3.75 a volume; single copies, $1.25.

The Biochemical Journal. Edited for the Biochemical Society by W. M. BAYLISS and ARTHUR HARDEN. Subscription price, $5.25 a volume.

The Journal of Ecology. Edited for the British Ecological Society by FRANK CAVERS. Subscription price, $3.75 a volume; single copies, $1.25.

NOTE.—Prices on back volumes vary, and postage from London is charged on back volumes and single copies.

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS

THE SUMMER QUARTER

AT

THE UNIVERSITY OF CHICAGO

1914

THE Summer Quarter at the University of Chicago is the most largely attended of the year, more than three thousand, seven hundred students having registered in the summer of 1913. The University year is divided into quarters: the Autumn, Winter, Spring, and Summer, and students are admitted at the opening of each quarter, and also in certain designated courses at the beginning of the second term of the Summer Quarter.

The courses during the Summer Quarter are the same in character, method, and credit value as in other quarters of the year. Students may enter for either term or both. The quarter will begin on June 15 and end August 28.

A large proportion of the regular faculty of the University, which numbers about three hundred and fifty, and also many instructors from other institutions, offer courses in the Summer Quarter, and in this way many varied points of view are given to students in their chosen fields of study.

ARTS, LITERATURE, AND SCIENCE

The University offers during this quarter, in the Schools of Arts, Literature, and Science, both graduate and undergraduate courses in Philosophy, Psychology, and Education; Political Economy, Political Science, History, Sociology and Anthropology, and Household Administration; Semitics and Biblical Greek; Comparative Religion; History of Art, Sanskrit, Greek, and Latin; Modern Languages; Mathematics, Astronomy, Physics, and Chemistry; Geology and Geography; Botany, Zoölogy, Physiology, Anatomy, Bacteriology; and Public Speaking.

THE PROFESSIONAL SCHOOLS

Education

In the Professional Schools the Graduate Department of Education in the School of Education gives advanced courses in Principles and Theory of Education, Educational Psychology, the Psychology of Retarded and Subnormal Children, History of Education, and Social and Administrative Aspects of Education. The College of Education is a regular college of the University, with all University privileges, and in addition provides the professional training of elementary- and secondary-school teachers and supervisors. It offers undergraduate courses in professional subjects and in the methods of arranging and presenting the various subject-matters which are taken up in the elementary and secondary schools. The University High School, with the fully equipped shops of the Manual Training School, is in session during the Summer Quarter, and opportunity is offered to take beginning courses in Latin and German and to review courses in Mathematics and History. The regular shop work supplemented by discussions of methods is open to teachers pursuing these courses.

Divinity

The Divinity School is open to students of all denominations, and the instruction is intended for ministers, missionaries, theological students, Christian teachers, and others intending to take up some kind of religious work. The English Theological Seminary, which is intended for those without college degrees, is in session only during the Summer Quarter. The Graduate Divinity School is designed primarily for college graduates. Pastors, theological teachers, students in other seminaries, and candidates for the ministry with requisite training are admitted in the Summer Quarter.

Law

In the work of the Law School the method of instruction employed—the study and discussion of cases—is designed to give an effective knowledge of legal principles, and to develop the power of independent legal reasoning. The three-year course of study offered constitutes a thorough preparation for the practice of law in any English-speaking jurisdiction. By means of the quarter system students may be graduated in two and one-fourth calendar years. Regular courses of instruction

counting toward a degree are continued through the Summer Quarter. The courses are so arranged that students may take one, two, or three quarters in succession in the summer only, before continuing in a following Autumn Quarter. The summer work offers particular advantages to teachers, to students who wish to do extra work, and to practitioners who desire to study special subjects.

Medicine

Courses in Medicine constituting the first two years of the four-year course in medicine of Rush Medical College are given at the University of Chicago. For the majority of students taking up medical work for the first time, it is of decided advantage to enter with the Summer or the Autumn Quarter. For the student who is lacking in any of the admission courses, or who seeks advanced standing, it is of especial advantage to enter for the Summer Quarter. All the courses offered are open to practitioners of medicine, who may matriculate as unclassified or as graduate students. Practitioners taking this work may attend the clinics at Rush Medical College without charge.

The University of Chicago is peculiarly fortunate in its environment in summer. The city of Chicago is relatively cool. High temperatures are not frequent or long continued, and the normal temperature, in comparison with that of other large cities, is low. Reports of the United States Weather Bureau show that the average summer temperature of Chicago is lower than that of most cities of its class. In addition to this advantage in weather conditions, the University has an especially favorable situation in the city. To the south stretches the Midway Plaisance, an avenue of lawn a block wide and a mile long; and about equidistant are Washington Park, a large recreation ground on the west, and Jackson Park, equally spacious, on the shore of Lake Michigan, to the east.

Opportunities for diversion are numerous. In Jackson Park there are golf links, and in both Jackson and Washington parks, lagoons for rowing. There are many tennis courts in both parks, along the Midway, and on the campus. Through the Frank Dickinson Bartlett Gymnasium and the Lexington Gymnasium full facilities for physical culture are given to both men and women. In the Bartlett Gymnasium

there is a swimming-pool open daily to men, and at designated hours in the week, to women. The Reynolds Club offers social privileges to men, and the Women's Union serves the social needs of women resident in the University. Two of the special attractions at the University are the beautiful new Harper Memorial Library, with its great reading-room and book collections open to the student, and the athletic field of the University, with its splendid new concrete grandstand seating nearly 10,000 people.

Notable public libraries and museums, highly organized industrial plants, many typical foreign colonies, a large number of settlements, and other significant social institutions make Chicago a peculiarly appropriate center for study and investigation.

A series of public lectures in Literature, History, Sociology, Science, Art, Music, etc., scheduled at late afternoon and evening hours throughout the Summer Quarter, affords an opportunity to students and other members of the University community to hear speakers of authority and distinction in many departments of study and activity. This program will include a number of popular readings and recitals, a series of open-air plays, a series of evening concerts, and excursions to places and institutions of interest in and near Chicago.

The complete ANNOUNCEMENT of courses for the Summer Quarter of 1914 will be issued in March, 1914, and may be obtained by application to

THE UNIVERSITY OF CHICAGO
CHICAGO, ILLINOIS

Recent Publications

OF

The University of Chicago Press

Unpopular Government in the United States. *By Albert M. Kales, Professor of Law in Northwestern University.*

272 pages, 12mo, cloth; $1.50, postage extra (weight 24 oz.)

This volume by a prominent member of the Chicago bar is an especially timely book, presenting with great clearness and cogency some of the political needs of the country, particularly the necessity of the short ballot. The author defines unpopular government as one of centralized power which is able to maintain itself in the face of popular disapproval. He then points out that the establishment in the United States of state and municipal governments, according to the plan of splitting up the power of government among many separate offices and requiring the widest and most frequent use of the elective principle, has cast so great a burden upon the electorate that an intelligent citizen is reduced to a state of political ignorance inconsistent with self-government. This situation has made it possible, he thinks, for a well-organized hierarchy to acquire the real power of government and to retain it, in the face of popular disapproval, for selfish ends. Such leaders the author characterizes as "politocrats."

The first part of the volume deals with the rise of the politocrats; the second discusses various expedients for restoring the American ideal of democracy; while the third considers constructive proposals like the commission form of government for smaller cities, and the application of the principles underlying this form to larger cities and the state, and to the selection of judges.

Chicago Tribune. Albert M. Kales, Professor of Law in Northwestern University, has written a book which ought to be read wherever citizens are perplexed by the intricacies and distressed by the failures of government.

NEW POSTAGE RATES

A change in the post-office regulations, effective March 16, 1914, provides for the mailing of books under the parcel-post classification. It is no longer possible to announce definite postage charges because of the variation of the rate with the distance.

We shall, however, still publish on the "net" basis, which involves a separate charge for transportation. In the following announcements the weight of each book is given, which will enable the purchaser to estimate the postage cost to his place of residence.

The French Verb: Its Forms and Tense Uses. By William A. Nitze and Ernest H. Wilkins, of the Department of Romance Languages and Literatures, the University of Chicago.

46 pages, 8vo, paper; 25 cents, postpaid 28 cents
French Verb Blanks, per pad 25 cents; postage extra (weight 14 oz.)

This little book prepared by Professor Nitze and Professor Wilkins, of the Department of Romance Languages and Literatures in the University of Chicago, will be of especial interest and value to all students and teachers of French because of the great help it gives in the mastery of the French verb. It is intended to facilitate familiarity with the verb forms and fix them in the student's memory by associating those tenses which are actually related in form. A method of classroom drill is suggested; the table and discussion of tense uses are self-explanatory; and the illustrative material, when quoted, is drawn from standard nineteenth-century writers.

Verb blanks, for the suggested arrangement of verb forms, will be supplied in pads at the prices indicated above.

The pamphlet is intended for use in both elementary and advanced courses, and is believed to be in method a great improvement over anything previously published.

Chicago and the Old Northwest, 1673–1835. By Milo Milton Quaife, Superintendent of the Wisconsin State Historical Society.

488 pages, 8vo, cloth; $4.00, postage extra (weight 46 oz.)

This book recounts, in a manner at once scholarly and dramatic, the early history of Chicago. Important as this subject is, it is not treated solely for its own sake. The author's larger purpose has been to trace the evolution of the frontier from savagery to civilization. From the point of view of Chicago and the Northwest alone the work is local in character, although the locality concerned embraces five great states of the Union; in the larger sense its interest is as broad as America, for every foot of America has been at some time on the frontier of civilization. It is believed that this book will take rank as the standard history of Chicago in the early days.

The Nation. In this monograph [on the history of Fort Dearborn] we have one of the most careful studies in Western history that has ever been made.

PUBLICATIONS OF THE CHICAGO HISTORICAL SOCIETY

The University of Chicago Press has become the publishing and distributing agent for the following books of the Chicago Historical Society:

Masters of the Wilderness. By Charles B. Reed. ("Fort Dearborn Series.")

156 pages, 16mo, cloth; $1.00, postage extra (weight 12 oz.)

In reproducing these romantic episodes of our exploration era the author has neither exaggerated the color nor distorted the facts of that intensely human period. The opening essay, which gives its title to the volume, is a highly interesting and carefully wrought account of the origin and upgrowth of the Hudson's Bay Company, with a portrayal of its powerful influence on the development of Canada. "The Beaver Club," the second essay in the volume, is closely allied to the first, and concerns a social club of Montreal the members of which were drawn from the partners and factors of the Northwestern Fur Company, for many years a rival of the Hudson's Bay Company. For forty years this club dominated the commercial, political, and social life of Canada. The concluding essay, "A Dream of Empire," recounts with many fresh details the adventures of Tonty in Old Louisiana.

The book as a whole is a successful attempt to awaken interest in some of the remarkable episodes of our early history. It is not analytical but narrative, not a sequence of annals but a series of picturesque activities.

COLLECTIONS

Vol. I. *History of the English Settlement in Edwards County, Illinois.* By GEORGE FLOWER. 402 pages, royal 8vo, cloth; out of print.

Vol. II. *Biographical Sketch of Enoch Long, an Illinois Pioneer.* By HARVEY REID. 134 pages, royal 8vo, cloth; $1.00, postage extra (weight 22 oz.).

Vol. III. *The Edwards Papers.* Edited by E. B. WASHBURNE. 662 pages, royal 8vo, cloth; $3.50, postage extra (weight 4 lbs.).

Vol. IV. *Early Chicago and Illinois.* Edited by EDWARD GAY MASON. 538 pages, royal 8vo, cloth; $3.00, postage extra (weight 57 oz.).

Vol. V. *The Settlement of Illinois.* By ARTHUR CLINTON BOGGESS. 268 pages, royal 8vo, cloth; $2.00, postage extra (weight 36 oz.).